OPEN ACCESS
Jurnal Ilmiah Ilmu Terapan Universitas Jambi
Vol. 9 No. 2. June 2025, pp. 578-597 DOI. 10.22437/jiituj.v9i2.42976
Research Article

TRANSFORMATION OF MATHEMATICAL KNOWLEDGE FOR TEACHING NON-EUCLIDEAN GEOMETRY CONCEPT THROUGH E-LEARNING BASED ON THE THEORY OF DIDACTIC SITUATIONS USING A MULTIPHASE MIXED METHOD

Mery Noviyanti[1,*], Sudirman[1], Camilo Andrés Rodríguez-Nieto[2]

[1] Universitas Terbuka, Banten, Indonesia
[2] Department of Exact Sciences, University of the Coast, Colombia
Corresponding author email: meryn@ecampus.ut.ac.id

Article Info
Received: Jan 12, 2025
Revised: Feb 24, 2025
Accepted: Apr 18, 2025
OnlineVersion: Apr 18, 2025

Abstract
This study aims to explore how mathematical knowledge related to non-Euclidean geometry can be transformed and improved through e-learning based on Theory of Didactic Situations (TDS). The method used is multiphase mixed method (MMM) involving around 20 mathematics teachers, consisting of 6 males and 14 females. Data collection was carried out through content development guidelines, observation sheets, interviews, and tests, and analyzed qualitatively and quantitatively. The results showed that TDS-based e-learning effectively improved teachers' understanding of non-Euclidean geometry, increased their engagement, and encouraged independent exploration during the learning process. However, challenges were still found in understanding hyperbolic, spherical, and elliptic geometry due to limited understanding of non-Euclidean axiomatic systems. The novelty of this study lies in the integration of the TDS approach in e-learning as an effective strategy to overcome obstacles in learning non-Euclidean geometry conceptually and visually.

Keywords: E-learning, Mathematical knowledge for teaching, Multiphase mixed method, Non-Euclidean geometry, Theory of didactic situations

INTRODUCTION

Geometry is a fundamental concept that mathematics teachers must master, as it plays a significant role in developing students' critical thinking, problem-solving, and logical reasoning skills (Alhogbi et al., 2018; Elshabasy et al., 2021; Fauziyah et al., 2023). Within the broader field of geometry, non-Euclidean geometry stands out as a complex yet intellectually stimulating topic (O'Rourke et al., 2020; Fitriani et al., 2023; Lagones & Alonso Ishihara, 2024; Ruiz & Gallagher, 2025). It offers a unique way to challenge long-held assumptions rooted in Euclidean thinking and encourages students and teachers alike to explore new dimensions of spatial reasoning (Mafarja et al., 2022; Jumaera et al., 2024; Rachmanto & Akande, 2024; Suroso et al., 2024). Mastery of this concept not only enriches mathematical understanding but also deepens the appreciation for the theoretical foundation of space, logic, and abstraction.

The teaching of non-Euclidean geometry presents both challenges and opportunities in mathematics education. For teachers, the abstract nature and unfamiliar axioms of non-Euclidean systems—such as hyperbolic, elliptic, and spherical geometry—can be difficult to convey using conventional instructional methods (Alaowffi & Alharbi, 2021; Setiawan et al., 2023; Worachak et al., 2023). Moreover, students often rely heavily on their prior understanding of Euclidean geometry, which can create cognitive dissonance when introduced to non-Euclidean principles (Bao, 2020; Lauc et al., 2020; Zulkhi, 2024). In order to facilitate more effective learning, there is a need for innovative teaching approaches that not only convey theoretical content but also promote conceptual understanding through engagement and visualization (Khadavi & Maulana, 2020; Eroglu, 2022; Mazmurrini et al., 2023).

Despite the recognized value of non-Euclidean geometry, research in mathematics education continues to focus predominantly on Euclidean concepts (Pajk et al., 2021; Hasibuan & Nugraha, 2023; Sari et al., 2023). Consequently, pedagogical strategies and instructional technologies specific to non-Euclidean geometry remain underexplored (Matias, 2020; Castellví et al., 2022; Kovalenko & Hontarenko, 2023). Many teachers, as highlighted by Tachie (2020) still struggle with foundational Euclidean concepts, which in turn hinders their ability to effectively introduce non-Euclidean topics. This underscores a significant gap in both teacher preparation and curriculum development, particularly concerning the integration of technology to support the learning process of non-Euclidean geometry.

Several studies have attempted to address this gap through innovative methods. For example, Ferrarello et al. (2019) investigated the use of GeoGebra software to visualize non-Euclidean structures. Ceramic panel designs inspired by Escher-style tessellations to teach symmetry (Hall et al., 2019). supported by Kotarinou and Stathopoulou's (2012) study exploring drama-based learning, while Although this method offers a creative alternative, the use of structured didactic frameworks such as the Theory of Didactic Situations (TDS) in e-learning environments has not been adequately investigated.

The novelty of this study lies in the integration of the Theory of Didactic Situations (TDS) into an e-learning platform to transform mathematical knowledge, specifically in teaching non-Euclidean geometry to mathematics teachers. While previous approaches have focused on creative tools or media, this study emphasizes the application of didactic theory that structures the learning environment around purposeful interactions and problem-based exploration. The Theory of Didactic Situations provides a theoretical framework that allows teachers to engage in autonomous learning and conceptual construction, supported by dynamic and visual digital resources (Alhassan et al., 2023; Hinojosa et al., 2020; Siddique et al., 2025). This approach not only improves understanding but also addresses common conceptual and procedural errors associated with the axiomatic structure and visualization of non-Euclidean spaces (Cebesoy & Yeniterzi, 2016; Berenguer-Rico & Wilms, 2021; Angelia et al., 2023).

Therefore, this study aims to transform mathematics teachers' understanding of non-Euclidean geometry through an e-learning platform based on the Theory of Didactic Situations (TDS). Using a multiphase mixed methods (MMM) approach, the study involved three stages: (1) developing instructional content integrated with TDS principles; (2) conducting a quantitative analysis to measure teachers' understanding and engagement with non-Euclidean concepts; and (3) conducting a qualitative analysis to explore the types of errors teachers experience and how the platform addresses them. The results are expected to contribute to the development of effective instructional designs for non-Euclidean geometry in pre-service and in-service teacher education.

RESEARCH METHOD

This research adopted a multiphase mixed method (MMM) design, which included a qualitative phase, a quantitative phase, and another qualitative phase. The MMM design was selected because it integrated qualitative and quantitative methods in parallel, enabling a deeper understanding of the studied phenomenon (Rosanti et al., 2022; Djou et al., 2023; Engelschalt et al., 2024). Each phase served a distinct purpose, with the results of one phase informing or shaping the next, thereby ensuring a comprehensive analysis (Tulabut et al., 2018; Ugwuanyi, 2022; Ocktaviani et al., 2024). The research began with a qualitative phase focused on designing e-learning platform based on TDS. This design incorporated Situations of Action, Situations of Formulation, Situations of Validation, and Situations of Institutionalization (Madhankumar et al., 2021). Subsequently, the analysis proceeded to the quantitative phase, where the impact of the e-learning platform on mathematics teachers' knowledge of non-Euclidean geometry was assessed. The final phase involved a qualitative analysis through interviews and an examination of errors in solving non-Euclidean geometry problems. An overview of the MMM stages was described in Figure 1.

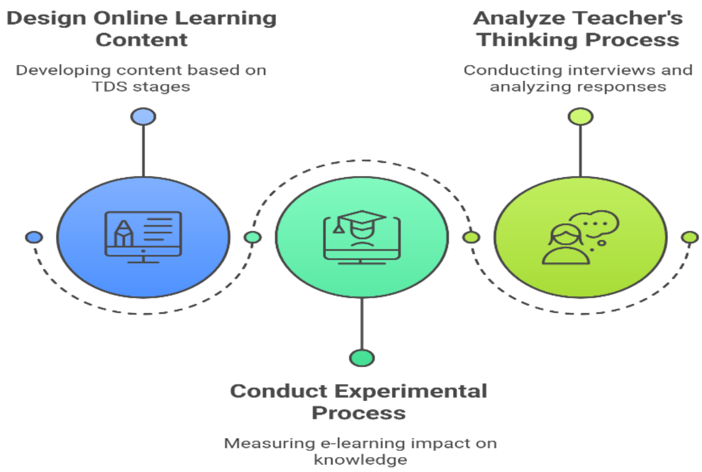

Figure 1. MMM stages

In this research, TDS (Brousseau, 1997) was integrated into e-learning platform to enhance the learning experience. During the Situations of Action phase, users independently explore concepts through initial materials and exercises, fostering foundational understanding. In the Situations of Formulation phase, users discussed and refined their ideas with friends or teachers through e-learning discussion forums. The Situations of Validation phase involved verifying understanding by comparing user-generated solutions with formal concepts. This could be facilitated through synchronous sessions on platforms such as Zoom or Microsoft Teams or by providing clear conclusions. Finally, in the Situations of Institutionalization phase, a confirmed concept was reinforced through assessments, particularly end-of-semester exams. The relationship between e-learning stages and TDS phases was presented in Figure 2.

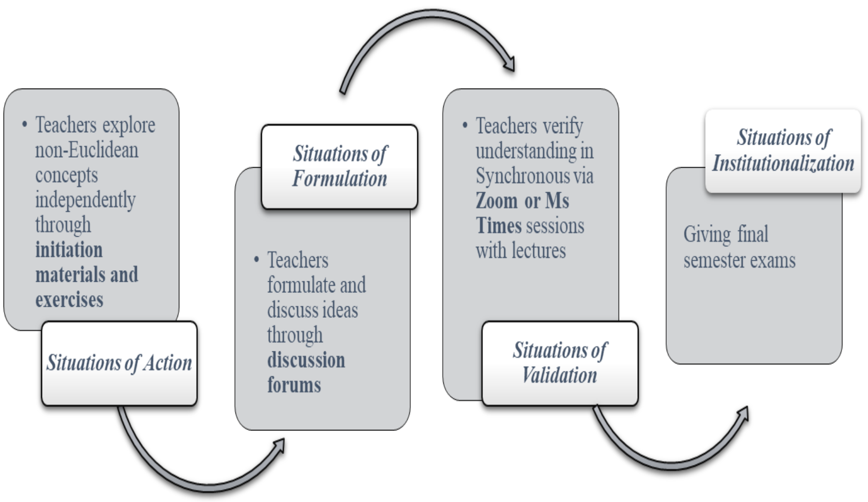

Figure 2. TDS integrated e-learning design framework

The integration of e-learning based on TDS followed a One-Group Pretest-Posttest Design, involving a single group of teacher participants. This program spanned seven sessions, each lasting two weeks (as detailed in Table 1). An illustration of the research implementation framework was provided in Figure 3.

Table 1. Context of Material and Time

No	Material	Time per Session	Source
1	Geometry Axioms	2 Weeks	https://elearning.ut.ac.id/ https://pustaka.ut.ac.id/reader/index.php
2	Affine Geometry	2 Weeks	elearning.ut.ac.id/ https://pustaka.ut.ac.id/reader/index.php
3	Projective Geometry	2 Weeks	elearning.ut.ac.id/ https://pustaka.ut.ac.id/reader/index.php
4	Inversive Geometry	2 Weeks	elearning.ut.ac.id/ https://pustaka.ut.ac.id/reader/index.php
5	Sphere Geometry	2 Weeks	elearning.ut.ac.id/ https://pustaka.ut.ac.id/reader/index.php
6	Hyperbolic Geometry	2 Weeks	elearning.ut.ac.id/ https://pustaka.ut.ac.id/reader/index.php
7	Elliptic Geometry	2 Weeks	elearning.ut.ac.id/ https://pustaka.ut.ac.id/reader/index.php

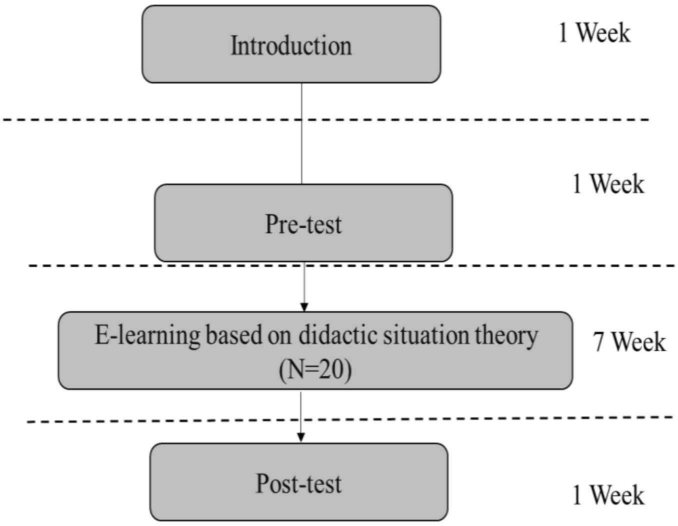

Figure 3. Implementation design

The sample in this study consisted of 20 mathematics teachers from junior and senior high schools in 39 provinces in Indonesia. Among the participants, 6 were male, and 14 were female, with an age range of 28 to 54 years. The sampling technique used was the purpose sampling technique. with the sampling criteria that teachers have diverse teaching experiences, offering various perspectives for analysis. The instruments used included content development guidelines, observation sheets, interview guidelines, and a mathematics knowledge test for teaching (MKT). The content development guidelines were designed to collaborate with the e-learning concept and TDS, which include Action Situations, Formulation, Validation, and Institutionalization (Ruiz & Gallagher, 2025). The guidelines aim to support the development of teachers' mathematical understanding of non-Euclidean geometry concepts (Saputra & Fahrizal, 2019). In addition, observation sheets were used to document the development process and interactions during the e-learning integration. The interview guidelines explored the types of errors made by teachers when solving non-Euclidean geometry problems. In the MKT test, pre-tests and post-tests were given to measure the improvement of teachers' understanding, as presented in Table 2.

Table 2. Indicators and Example Test Questions

No	Indicator	Example
1	Able to use the axiom of connection to prove related theorems.	Given a homogeneous coordinate system for the two-dimensional projective space $\mathbb{P}^{\mathbf{2}}$ with coordinates $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$. Suppose there is a projective transformation defined by the $\mathbf{3} \times \mathbf{3}$ matrix as follows: $\boldsymbol{P} = \begin{pmatrix} \mathbf{0} & \mathbf{1} & \mathbf{7} \\ \mathbf{4} & \mathbf{5} & \mathbf{3} \\ \mathbf{2} & \mathbf{-2} & \mathbf{2} \end{pmatrix}$ A line in the projective space is given by the homogeneous equation: $\mathbf{3}\boldsymbol{x} - \mathbf{2}\boldsymbol{y} + \boldsymbol{z} = \mathbf{0}$. Determine the image of this line under the projective transformation given by the matrix $\boldsymbol{P}$. Explain the steps used to determine the image of the line and interpret the result in the context of projective geometry.
2	Able to use the fundamental theorems of Affine Geometry to prove Ceva's Theorem.	
3	Able to determine the image of a line under a projective transformation.	

The data obtained from interviews and observations were analyzed using content analysis consisting of several stages. These stages included raw data collection, data understanding, open coding, axial coding, theme refinement, and theme reporting, as detailed in Figure 4.

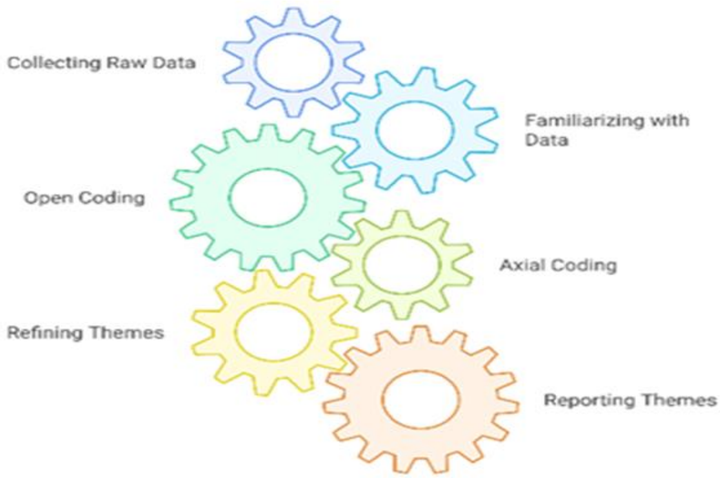

Figure 4. Content Analysis Process

The research process began with the collection of raw data from e-learning activities, including discussion outcomes and assignments. The data was then analyzed in several steps. First, the analysts became familiarized with the data to identify initial patterns. This was followed by open coding, where specific units such as statements or actions were assigned codes. Subsequently, axial coding was applied to uncover relationships between categories. The main themes were refined by aligning the data with the identified categories, and the final themes were systematically reported to provide comprehensive insights into the effectiveness of TDS-based e-learning design and implementation. Furthermore, descriptive statistics were used to analyze data from assignments and discussions in sessions 1–12, with the average final scores calculated. Test data was analyzed using a one-sample t-test to measure changes within the group, controlled for individual variation, and assessed the impact of the e-learning based on TDS.

Before participating, all teachers received detailed information about the research objectives, procedures, benefits, and potential risks. Participation was voluntary, and teachers provided informed consent by signing a consent form. Confidentiality and anonymity were maintained by assigning identification codes and securely storing data in password-protected digital archives accessible only to the analysts. Participants had the right to withdraw from the analysis at any point without consequences.

To ensure minimal risks, the research offered flexibility in scheduling surveys and interviews. All collected data was securely stored and would be deleted after a predetermined retention period. Furthermore, the research obtained approval from the relevant ethical review board, adhering to ethical standards in educational contexts. Throughout the analysis, participants were treated fairly and respectfully, without discrimination based on gender, ethnicity, or background. The contributions of these participants were valued and acknowledged.

RESULTS AND DISCUSSION

E-Learning Design Based on TDS

The e-learning platform used in this research was based on the Learning Management System (LMS) provided by Universitas Terbuka. It was designed to facilitate online access to lecture materials, assignments, exams, and interactions with teachers and friends. This e-learning system supported diverse learning methods, including lecture videos, discussion forums, online quizzes, and digital teaching resources, which were accessible at any time. An illustration of the platform's interface was shown in Figure 5.

Figure 5. Front view of e-learning

The platform's login page required users to enter their username and password to access the system. Once logged in, teachers were directed to the main dashboard, which featured various learning tools. The dashboard allowed users to manage course materials, create quizzes, and monitor students' progress. Additionally, it included a Discussion Forum for interactive communication between teachers and users and provided access to Online Tutorial Accounts, simplifying the management of learning activities and communication with users.

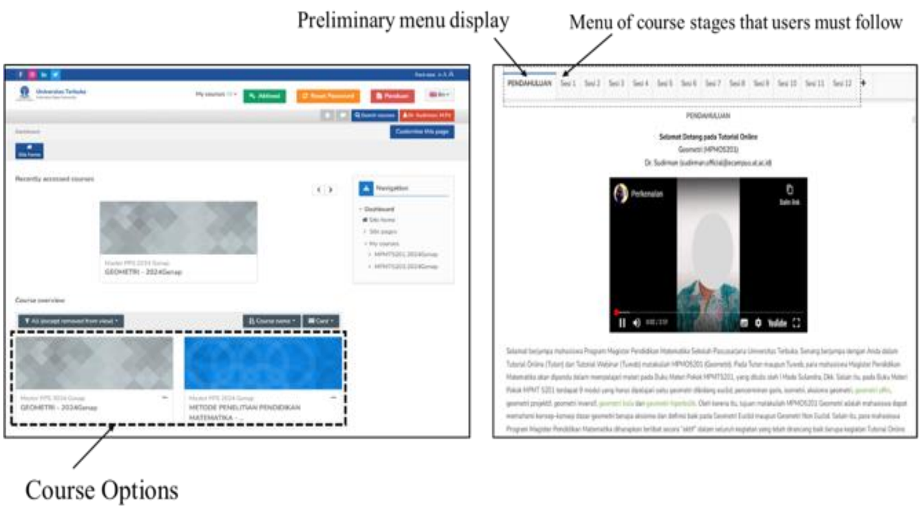

Figure 6. Course Menu and Main Menu Display

After logging in, users could select the courses they wish to access, as shown in Figure 6. For instance, upon selecting the "Geometry" course, users were presented with a menu and a sequence of activities organized into an introductory section and sessions 1 through 12. The introductory section provided essential information, including course objectives, material indicators, session overviews, activity guidelines, rules, reading resources for each session, and instructions for face-to-face virtual sessions conducted through Zoom.

Before proceeding with the activities outlined for each session, users must first review the materials available in the "Geometry" teaching module. These materials could be accessed through the link: https://pustaka.ut.ac.id/lib/mpmt5201-geometri-suplemen/. The teaching module included comprehensive materials, exercises, and formative tests for users to complete. The virtual reading interface for these materials was detailed in Figure 7.

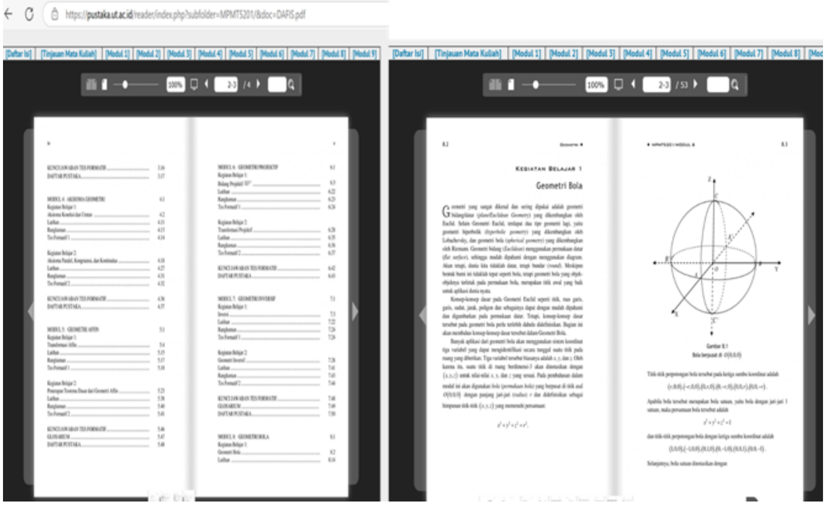

Figure 7. Virtual Reading Room for Geometry Material

Upon completing the review of a given chapter, users proceeded to engage with the corresponding session activities on the e-learning platform. For instance, after studying Chapter 1 in the teaching materials, users began Session 1 within the e-learning system. This step-by-step method ensured a structured learning experience, seamlessly integrating teaching materials with the activities outlined on the platform.

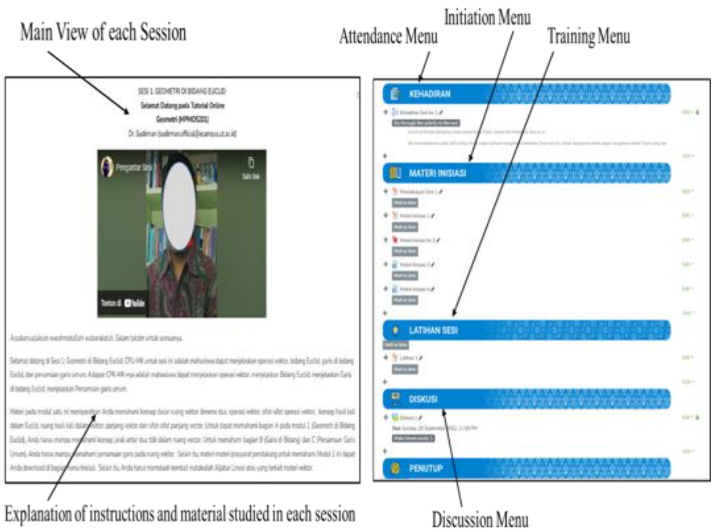

Figure 8. Core View of Each Session

In the core view of the e-learning platform, 6 menus were available, including Greeting, Attendance, Initiation Material, Practice Questions, Discussion, and Closing (Figure 8). Additionally, direct interactions were conducted during sessions 3, 6, 9, and 12 through Zoom or Microsoft Teams. The e-learning design, which included initiation materials and practice exercises derived from both teaching materials and other resources, aimed to provide participants with an independent understanding of initial concepts. These initiation and practice materials supported the Situations of the Action stage. In the Situations of Formulation stage, users were encouraged to formulate ideas and discuss concepts they had learned. This was facilitated through the Discussion menu, as detailed in Figure 9.

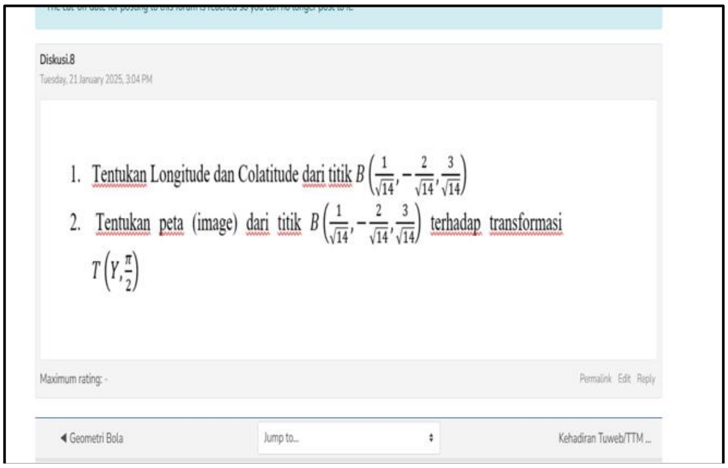

Figure 9. Discussion Menu

During the Situations of Validation stage, users verified their understanding by comparing their answers or solutions with formal concepts. This stage was supported by live sessions conducted through Zoom or Microsoft Teams with teachers (Figure 10). Lastly, in the Situations of Institutionalization stage, the concept that have been verified and solidified become part of broader knowledge. This was reinforced through access to online teaching materials available in the virtual reading room, which served as a learning reference.

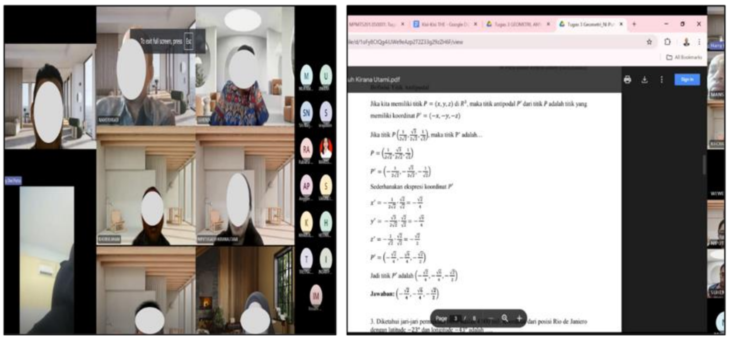
Figure 10. Synchronous Process

Impact of E-learning based on TDS

To evaluate the effectiveness of the learning process, this research assessed the participants' performance in discussions. Discussion topics were designed to include problem-based questions that participants were required to answer. An example of such a question was shown in Figure 9. Each participant submitted their answers to the discussion forum, where other participants could respond to the answers provided. Teachers then assessed the participants' responses, and the performance data from discussions conducted in sessions 1 to 12 was presented in Table 3.

Table 3. Average result data from Discussion

Discussion	Participants	Average Score	Standard Deviation Score
1	20	85.8	5.74
2	20	79.75	8.74
3	20	85.15	2.21
4	20	88.4	1.57
5	20	84.74	1.88
6	20	85	1.33
7	20	84.8	0.89
8	20	91.5	1.73
9	20	84.25	1.48
10	20	84.25	1.07
11	20	89.2	2.57
12	20	88.85	2.54

The analysis of Table 3 showed that participants' performance in discussions was relatively stable, with average scores ranging from 79.75 to 91.5. The lowest average score occurred during the second discussion session, which also had the highest standard deviation (8.74), suggesting significant variation in participants' understanding. Conversely, the eighth discussion recorded the highest average score (91.5) with a low standard deviation, showing a strong understanding of the material by participants in that session. In summary, participants' scores improved and stabilized between the fourth and eighth discussions, though slight fluctuations were observed in the later sessions. The relatively low standard deviations across most sessions showed that participants' understanding was generally consistent, even though certain topics proved to be more challenging. This showed participants' progress throughout the discussions. The trend in discussion scores was showed in Figure 11.

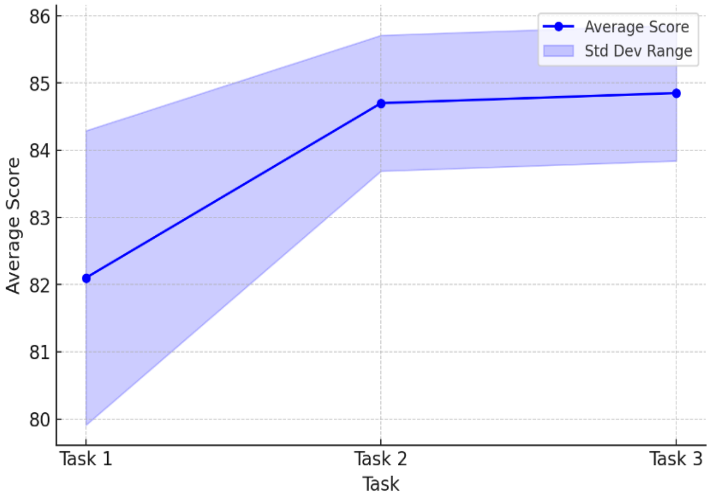

Figure 11. Trend of Discussion Score Development

Data were analyzed from tasks 1, 2, and 3, focusing on the number of participants, average task scores, and standard deviations. The results of this analysis provided insights into the development of participants' performance across different tasks.

Table 4. Average scores for tasks 1, 2 and 3

Task	N	Average assignment score	Standard deviation of task scores
1	20	821.	2.19
2	20	84.7	1.01
3	20	84.85	1.01

From Table 4, it was evident that the average task scores increased progressively from Task 1 to Task 3. The average score for Task 1 was 82.1, with a standard deviation of 2.19, showing a slightly higher variation in participants' performance compared to subsequent tasks. In contrast, Tasks 2 and 3 showed improved average scores of 84.7 and 84.85, respectively, accompanied by a lower standard deviation of 1.01. This suggested more consistent scores among participants, as detailed in Figure 12.

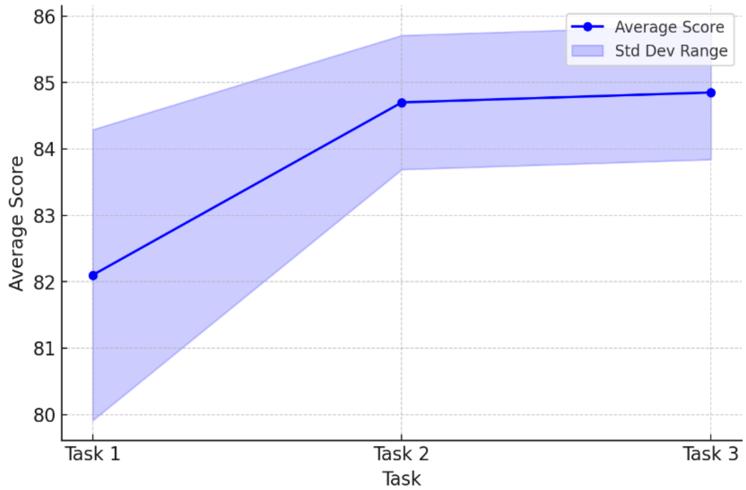

Figure 12. Trends in Task Scores

After the completion of 12 learning sessions, participants were administered a post-test to evaluate their understanding of the studied material. This post-test aimed to assess the extent of improvement in participants' skills after completing learning process. The descriptive statistics of the post-test results were presented in Table 5.

Table 5. Descriptive Statistics

	N	Mean	Std. Deviation	Std. Error Mean
Post-test Results	20	83.5000	11.65965	2.60718

The analysis of the post-test data showed an average score of 83.50 among the 20 participants, showing a generally high level of achievement well above the standard threshold of 65. However, the standard deviation of 11.66 highlighted considerable variation in scores, reflecting differing levels of understanding among participants. The standard error of the mean, calculated at 2.61, showed a reasonable level of accuracy in estimating the population mean, with minimal potential for large fluctuations. While the average score suggested strong performance, the substantial variation in scores underscored the need for further evaluation of individual participants understanding.

Table 6. Tests of Normality

	Shapiro-Wilk		
	Statistic	df	Sig.
Posttest Results	.952	20	.405

Table 7. One-Sample Test

	Test Value = 65					
					95% Confidence Interval of the Difference	
	T	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
Posttest Results	7.096	19	.000	18.50000	13.0431	23.9569

The results of the Shapiro-Wilk normality test (Table 6) confirmed that the data followed a normal distribution ($p = 0.405$). Consequently, a one-sample t-test (Table 7) was conducted to compare the class average to the standard benchmark of 65. The t-test results yielded a t-value of 7.096 with $p < 0.001$, showing that the class average was significantly higher than the standard benchmark. This showed that the participants' understanding of non-Euclidean geometry was statistically superior to the minimum expected level, classifying their comprehension as highly satisfactory.

Error Analysis After Implementation of E-learning based on TDS

Despite most participants achieving scores above the benchmark of 65, an error analysis showed recurring mistakes in two specific topics, including hyperbolic and spherical geometry. Figure 13 presented the types of errors identified in these areas.

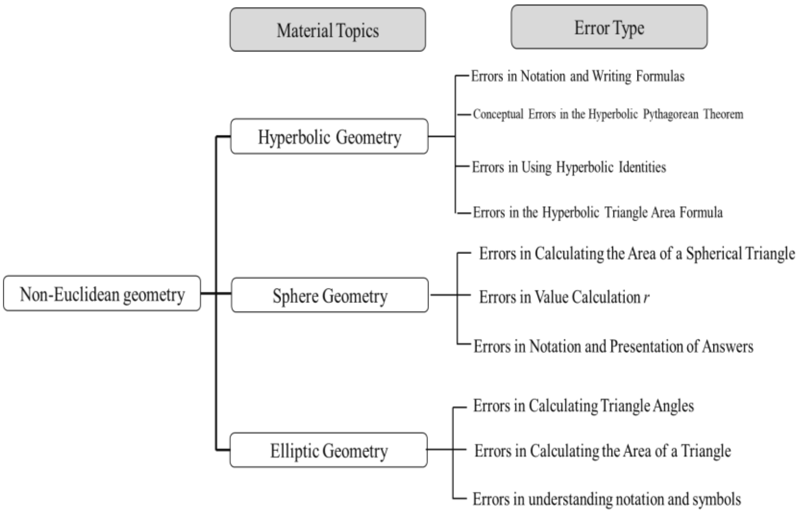

Figure 13. Types of errors

In solving hyperbolic geometry problems such as those in Figure 13, there were several types of errors that could be identified in solving the hyperbolic geometry problems.

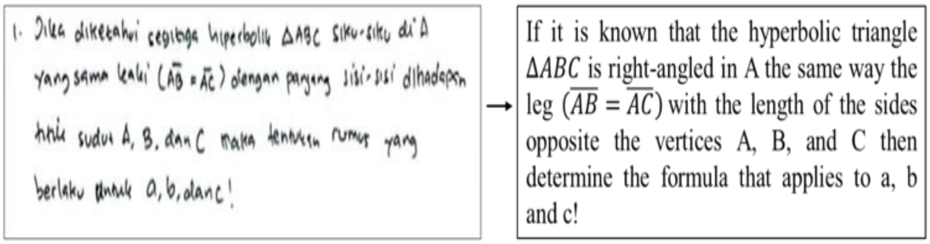

Figure 14. Hyperbolic Geometry Problems

When solving the problem, some participants often use the notation of the hyperbolic geometry mode incorrectly. Some of the notations used seem confusing and inconsistent. In the section on using the hyperbolic Pythagorean theorem (see Figure 14), there was inconsistency in the notation. For instance, the symbol such as $cosh^2(c) = cosh^2(a)$ was used without a clear context. The symbol $cosh^b(c) = cosh^b(a)$ was not written correctly and could confuse readers because there was no clear rule regarding the exponent in hyperbolic geometry. Furthermore, the steps involved in calculations and transformations of equations did not always show how one step transitioned to the next, leading to logical errors. The next error was related to the conceptual error in the hyperbolic Pythagorean theorem. While the Pythagorean theorem in Euclidean geometry stated $c^2 = a^2 + b^2$ for a right triangle, in hyperbolic geometry, this relationship changed to $cosh(c) = cosh(a) + cosh(b)$. Conceptual errors occurred when students miswrite or misinterpret this formula. The answer might state that "because it was a right angle, the Pythagorean theorem applied to hyperbolic triangles" but the transformations performed did not follow the correct rules. In the step $cosh^2(c) = cosh^2(a)$, the form that complied with the hyperbolic rules should be used, namely $cosh(c) = cosh(a)cosh(b)$ instead of the standard quadratic form (See Figure 15). Errors often occur when students use hyperbolic identities in an inappropriate way or do not match the conditions given in the problem. For instance, the use of $cosh^2(c) = cosh^2(a)$ without explaining its mathematical basis. The error in the simplification

$cosh(c) = cosh(a)cosh(b)$ should obey the basic form of the hyperbolic identity, which was not always true without additional conditions. The hyperbolic identities used in the calculation steps needed to be further clarified and corrected.

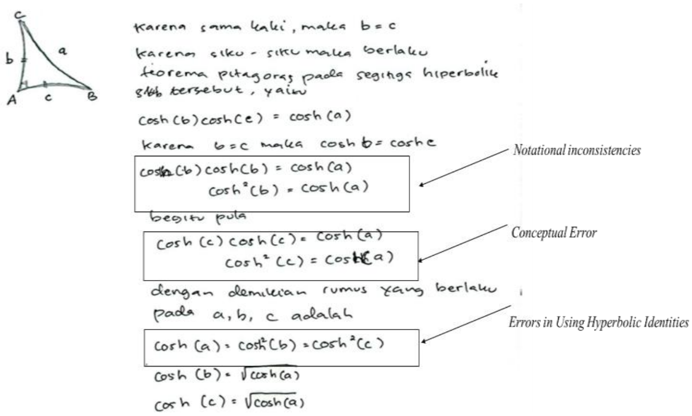

Figure 15. One of the participant's answers

The next error was the error in using the hyperbolic triangle area formula. The area of a triangle in Euclidean geometry was calculated using the formula $L\Delta = \frac{1}{2}$ base x height. However, in hyperbolic geometry, the area of a triangle was given by $L\Delta = \pi - (A + B + C)$. The error occurred when the participants tried to adjust this formula without considering the conditions of the triangle given. For instance, an answer stated $\Delta ABC = \pi - (A + 2B)$, which did not match the general formula for the area of a hyperbolic triangle. This error showed students were yet to understand the concept that the sum of the angles in a hyperbolic triangle was always less than π, as detailed in Figure 16.

Figure 16. Error in using the hyperbolic triangle area formula

Next, when participants are given problems as in Figure 17.

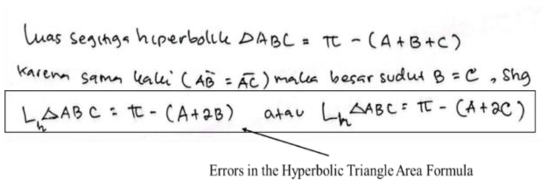

Figure 17. Second problem

In hyperbolic geometry, the formula for the area of a triangle was $L\Delta = \pi - (A + B + C)$, where A, B, and C were the interior angles of the hyperbolic triangle. The answer in the image ($\Delta = \pi - (A + \alpha + \beta)$) used this formula correctly, but there was an error in the angle substitution. In fact, in a right triangle at A, angle A should already be known as $\frac{\pi}{2}$, making the correct formula to be $L\Delta = \pi - (\frac{\pi}{2} + \alpha + \beta)$. The correction needed was to use the value $A = \frac{\pi}{2}$ from the beginning to get the correct result, namely $L\Delta = \pi - (\frac{\pi}{2} + \alpha + \beta)$ or $L\Delta = \frac{\pi}{2} - (\alpha + \beta)$. Also, in hyperbolic geometry, the sum was always less than π. The error in the answer stated that "a right angle in hyperbolic geometry was less than 90^o atau $\frac{\pi}{2}$". This was not true in hyperbolic geometry, where a right angle was still 90^o atau $\frac{\pi}{2}$, but the sum of the angles of a triangle was less than 180° or π. The correction needed was to state that $A = \frac{\pi}{2}$ without doubting its size because the property of right angles was still applied in hyperbolic geometry (See Figure 18). Another error that was observed in the answer was the way it was written and explained concept. The notation and thought process were not perfectly explained, making it difficult to follow how the answer arrived at the result. It did not explicitly explain why $A = \frac{\pi}{2}$ must be directly substituted into the formula for the area of a triangle.

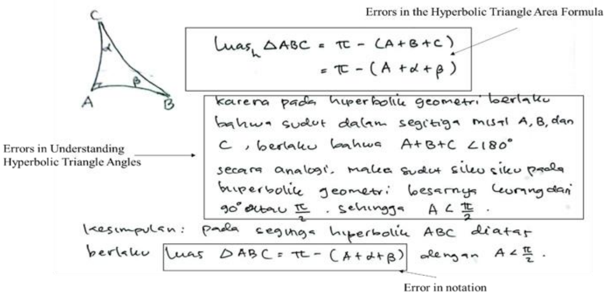

Figure 18. Errors in notation

Next, in solving problems related to spherical geometry (See Figure 19).

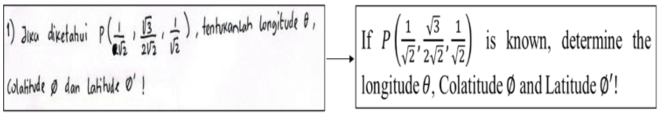

Figure 19. Spherical Geometry Problems

The participants made an error in calculating r, using the formula $r = \sqrt{x^2 + y^2 + z^2}$ with $x = \frac{1}{2\sqrt{2}}$, $y = \frac{\sqrt{3}}{2\sqrt{2}}$, $z = \frac{1}{\sqrt{2}}$. However, they were correct in writing $r = \sqrt{\left(\frac{1}{2\sqrt{2}}\right)^2 + \left(\frac{\sqrt{3}}{2\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$. Another error observed was the final result written as r =1, which should be $= \sqrt{\frac{11}{8}}$. The value of r being $\neq 1$ was considered an error in simplifying the result, as shown in Figure 20.

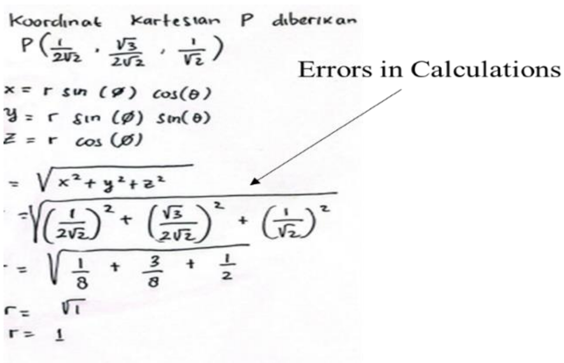

Figure 20. Error in calculation

Another error was observed in calculating Longitude θ. Longitude θ was calculated using $\theta = tan^{-1}\left(\frac{y}{x}\right)$. The answer showed $\theta = tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$, but the final result was written as θ = 60^{o} atau $\frac{\pi}{3}$. This showed that there was an error in the calculation process.

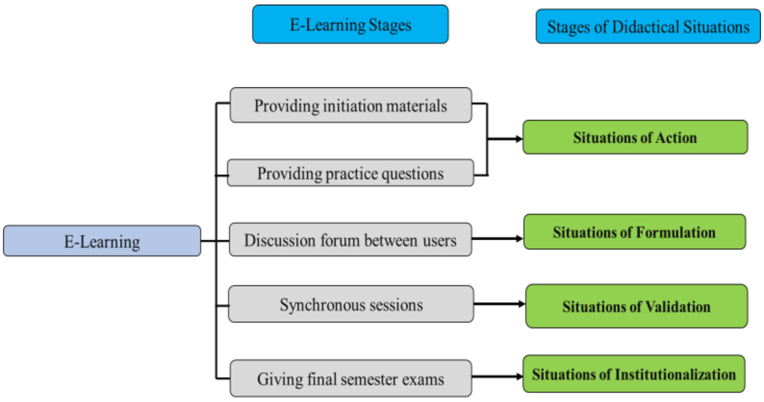

Figure 22. TDS-based e-learning design

The e-learning design developed in this study shows a close relationship with the stages in the Theory of Didactic Situations (TDS) as depicted in Figure 22. In the early stages, the material is presented in various formats—text, video, and other learning resources—which serve as stimuli to encourage students' independent exploration. This is in accordance with the “situation of action” stage in Brousseau's theory, where learners begin to understand concepts through initial interactions with the material. Showing that independent exploration in e-learning increases students' engagement and understanding of the material (Mulqueeny et al., 2015; Al-Doghan and Piaralal, 2024, Bayaga and Alexander,2023), support this finding by.

In the next stage, providing practice questions allows students to build a deeper conceptual understanding, in line with the “situation of formulation” stage. Discussion forums in e-learning also act as a medium for validating students' understanding, which reflects the “situation of validation” stage in TDS. In this forum, understanding is tested through argumentation and justification between participants. This finding is in line with the research results of Hew and Cheung (2014), and Lo and Hew (2020), which stated that online discussions can significantly improve students' conceptual understanding. Synchronous sessions, such as online lectures and live Q&A, have also been shown to support more effective validation of understanding than fully asynchronous learning (Martin et al., 2021; Romero-Hall, 2024; Lanawaang & Mesra, 2024). The last stage, namely the final exam, functions as a stage of "institutionalization," where the concepts that have been learned are integrated into students' cognitive frameworks to be applied in various contexts.

Quantitatively, the implementation of TDS-based e-learning showed a significant increase in understanding the concept of Mathematical Knowledge for Teaching (MKT) of non-Euclidean geometry. The t-test results showed a value of $t = 7.096$ with a significance of $\rho < 0.001$, indicating that the average class score was higher than the set benchmark. Which showed that blended e-learning was more effective than the fully asynchronous method (Yumiati et al., 2024; Mokoginta & Mokwena, 2024)). In addition, Moreno-Guerrero et al. (2020) and Shida et al. (2019) showed that e-learning can increase motivation, participation, and understanding of mathematical concepts through active engagement and metacognition. Interactive visualizations such as GeoGebra and personalization features in e-learning also support flexibility and independent learning according to the pace of each participant.

Various previous studies support these results, both in terms of effectiveness and challenges in implementing e-learning in learning non-Euclidean geometry. Showed that students' spatial intuition formed from Euclidean geometry experience is a major obstacle in understanding non-Euclidean concepts (Jones ,2002; Costa, 2017). The lack of interactive learning media as a factor that causes difficulties in understanding hyperbolic and elliptic geometry (Hadadi, 2018; Sukri et al., 2024). Emphasized the importance of physical and visual approaches in facilitating students' understanding of non-Euclidean space, such as the use of saddle or spherical surfaces. This shows that conventional teaching strategies have not fully addressed the conceptual challenges faced by students.

The novelty of this study lies in the comprehensive integration of e-learning design with the stages in TDS that are systematically applied to non-Euclidean geometry material. This study not only shows statistical effectiveness, but also reveals in depth how each stage of TDS can be contextualized in e-learning features such as independent exploration, discussion forums, and synchronous sessions. In addition, this approach is specifically applied to strengthening teacher MKT, which has not been widely explored in the realm of technology-based non-Euclidean geometry (Aisyah et al., 2023; Oktasari, 2024). The emphasis on the formulation and validation stages through digital interaction also shows a new contribution in creating an online collaborative learning space that encourages high-level thinking processes.

However, this study has limitations. Error analysis shows that although TDS-based e-learning successfully improves general understanding, there are still significant conceptual, procedural, and definitional errors, especially in understanding non-Euclidean axiomatic systems and space visualization. The implication of this finding is the need for further development of interactive media and visualizations that are able to represent the characteristics of non-Euclidean space more concretely. The implementation of problem-based learning strategies and exploratory approaches is also recommended to deepen students' learning experiences. Thus, innovation in instructional design remains the main key to improving non-Euclidean geometry literacy among both teachers and students.

CONCLUSION

The conclusion of this study shows that the developed e-learning design is in line with the TDS theory through the stages of action, formulation, validation, and institutionalization, and is statistically proven to improve teachers' understanding of non-Euclidean geometry MKT concepts ($t = 7.096$, $\rho < 0.001$). This e-learning is able to encourage motivation, engagement, and conceptual understanding through self-exploration features, online discussions, and synchronous sessions such as lectures and Q&A. However, errors were still found in understanding non-Euclidean geometry concepts, especially in hyperbolic, spherical, and elliptic geometry, due to a lack of understanding of the axiomatic system and difficulty in visualizing space. Therefore, it is recommended to use visual and interactive methods such as interactive visualization and simulation software, as well as the integration of problem-based and exploratory learning strategies to improve non-Euclidean geometry literacy as a whole.

ACKNOWLEDGMENTS

The researcher thanks Universitas Terbuka for providing financial support for the implementation of this research.

AUTHOR CONTRIBUTIONS

The first author was responsible for conceptualization, methodology, formal analysis, and writing (original draft preparation). The second author conducted a formal analysis, prepared resources, and handled visualization. The third author was responsible for validation, writing (review and editing), and supervision. The fourth and fifth authors were involved in validation and formal analysis.

CONFLICTS OF INTEREST

The author(s) declare no conflict of interest.

REFERENCES

Adelabu, F. M., Makgato, M., & Ramaligela, M. S. (2019). Enhancing learners' geometric thinking using dynamic geometry computer software. Journal of Technical Education and Training, 11(1), 44-53. https://publisher.uthm.edu.my/ojs/index.php/JTET/article/view/3092/2134

Aisyah, N., Susanti, E., Meryansumayeka, Siswono, T. Y. E., & Maat, S. M. (2023). Proving Geometry Theorems: Student Prospective Teachers’ Perseverance and Mathematical Reasoning. Infinity Journal, 12(2), 377–392. https://doi.org/10.22460/infinity.v12i2.p377-392

Alaowffi, N., & Alharbi, B. (2021). Preposition stranding under sluicing: Evidence from Hijazi Arabic. Journal of Language and Linguistic Studies, 17(2), 941–957. https://doi.org/10.52462/jlls.65

Al-Doghan, M. A., & Piaralal, S. K. (2024). Exploring the impact of e-learning initiatives on students' self-efficacy and e-learning experiences. Eurasian Journal of Educational Research, 109(109), 158-176. https://ejer.com.tr/manuscript/index.php/journal/article/view/1594

Alhassan, H., Al-Hassan, B., & Osei, M. (2023). Comparative Study of UCC and UEW B.Ed JHS Visual Art Options as Initial Teacher Preparation for the New Creative Arts and Design Curriculum for JHS. Journal Evaluation in Education (JEE), 4(4), 125–133. https://doi.org/10.37251/jee.v4i4.719

Angelia, A., Muminov, S. Q., & Obafemi, D. T. (2023). Development of Physics Modules with Science, Environment, Technology, and Society (SETS) Approaches to Motion and Force Material.

Schrödinger: Journal of Physics Education, 4(3), 78–85. https://doi.org/10.37251/sjpe.v4i3.694
Bao, W. (2020). COVID-19 and online teaching in higher education: A case study of Peking University. Human Behavior and Emerging Technologies, 2(2), 113–115. https://doi.org/10.1002/hbe2.191
Bayaga, A., & Alexander, M. (2023). Exploring the influence of adaptive e-learning technology on learners' perceptions of mathematical modeling and simulation in South African Schools. Computers in the Schools, 1-20. https://doi.org/10.1080/07380569.2023.2297893
Behnamnia, N., Kamsin, A., Ismail, M. A. B., & Hayati, A. (2020). The effective components of creativity in digital game-based learning among young children: A case study. Children and Youth Services Review, 116, 105227. https://doi.org/10.1016/j.childyouth.2020.105227
Benvenuti, S., & Cardinali, A. (2018). The mental telescope: understanding the geometry of Euclid by learning the non-Euclidean geometry. In Inted2018 Proceedings (Vol. 1, pp. 9296-9301). IATED. https://doi.org/10.21125/inted.2018.2287
Berenguer-Rico, V., & Wilms, I. (2021). Heteroscedasticity testing after outlier removal. Econometric Reviews, 40(1), 51–85. https://doi.org/10.1080/07474938.2020.1735749
Caerols, H., Carrasco, R. A., & Asenjo, F. A. (2021). Using smartphone photographs of the Moon to acquaint students with non-Euclidean geometry. American Journal of Physics, 89(12), 1079-1085. https://doi.org/10.1119/10.0006156
Castellví, J., Escribano, C., Santos, R., & Marolla, J. (2022). Futures education: Curriculum and educational practices in Australia, Spain, and Chile. Comunicar, 30(73), 45–55. https://doi.org/10.3916/c73-2022-04
Cebesoy, Ü. B., & Yeniterzi, B. (2016). 7th Grade Students' Mathematical Difficulties in Force and Motion Unit. Turkish Journal of Education, 5(1), 18. https://doi.org/10.19128/turje.51242
Chiu, T. K., & Hew, T. K. (2018). Factors influencing peer learning and performance in MOOC asynchronous online discussion forum. Australasian Journal of Educational Technology, 34(4), 16-28. https://doi.org/10.14742/ajet.3240
Costa, V. (2017). Geometric and intuitive space in Husserl. The Changing Faces of Space, 125-137. https://doi.org/10.1007/978-3-319-66911-3_8
De Freitas, S. I., Morgan, J., & Gibson, D. (2015). Will MOOCs transform learning and teaching in higher education? Engagement and course retention in online learning provision. British journal of educational technology, 46(3), 455-471. https://doi.org/10.1111/bjet.12268
Djou, D. N., Ntelu, A., & Hinta, E. (2023). Analysis of Code-Mixing in Marriage Proposals among Gorontalo People in Indonesia. Eurasian Journal of Applied Linguistics, 9(1), 48–58. https://doi.org/10.32601/ejal.901005
Elshabasy, M. M. Y. B., Al-Moghazy, M. A., & El Gamal, H. A. (2021). Investigation of the enhancement of microelectromechanical capacitive pressure sensor performance using the genetic algorithm optimization technique. Engineering Optimization, 53(1), 1–17. https://doi.org/10.1080/0305215X.2019.1694673
Engelschalt, P., Bielik, T., Krell, M., Krüger, D., & zu Belzen, A. U. (2024). Investigating pre-service science teachers' metaknowledge about the modelling process and its relation to metaknowledge about models. International Journal of Science Education, 46(7), 691–714. https://doi.org/10.1080/09500693.2023.2253368
Eroglu, S. (2022). A study of vocabulary in oral expressions of Indonesian students learning Turkish as a foreign language in terms of diverse variables. African Educational Research Journal, 10(1), 38–45. https://doi.org/10.30918/aerj.101.22.010
Fauziyah, A. N., Ramadan, M., Gumede, P. R., & Udosen, I. N. (2023). Development of Digital Book Bilingual Physics Learning Media Using Kvisoft Flipbook for High School Class X Semester 1 Subject of Newton's Law. *Journal of Educational Technology and Learning Creativity*, *1*(1), 7–15. https://doi.org/10.37251/jetlc.v1i1.618
Ferrarello, D., Mammana, M. F., & Taranto, E. (2019). Non-Euclidean Geometry with Art by Means of GeoGebra. *International Journal for Technology in Mathematics Education*, *26*(3), 113-119. https://doi.org/10.1564/tme_v26.3.02
Fitriani, F., Triandafillidis, T., & Thao, L. P. (2023). Exploring the Integration of Computational Thinking and Mathematical Modelling in STEM Education. *Interval: Indonesian Journal of Mathematical Education*, *1*(2), 73-82. https://doi.org/10.37251/ijome.v1i2.1341.
Hall, A., Brás, I., & Pais, S. (2019). Using an artistic approach to the teaching of non-Euclidean geometry in a professional development course for mathematics teachers. In INTED2019

International Technology, Education and Development Conference, Valencia, Spain, March 11-13, 2019 (pp. 2923-2930). IATED.

Hasibuan, L., & Nugraha, A. (2023). Development of Scientific and Constructivism-Based Handouts on Social Arithmetic Materials. Indonesian Journal of Education Research (IJoER), 4(2), 28–31. https://doi.org/10.37251/ijoer.v4i2.578

Hew, K. F., & Cheung, W. S. (2014). Students' and instructors' use of massive open online courses (MOOCs): Motivations and challenges. Educational research review, 12, 45-58. https://doi.org/10.1016/j.edurev.2014.05.001

Hinojosa, J., Martínez-Viviente, F. L., Garcerán-Hernández, V., & Ruiz-Merino, R. (2020). Teaching-learning model for the science of electronics. Journal of Technology and Science Education, 10(1), 87–100. https://doi.org/10.3926/jotse.604.

Hwang, G. J., Chu, H. C., & Yin, C. (2020). Objectives, methodologies, and research issues of learning analytics applications. Interactive Learning Environments, 28(2), 267-277. https://doi.org/10.1080/10494820.2018.1423132.

Jones, K. (2002). Issues in the teaching and learning of geometry. In Linda Haggarty (Ed.), Aspects of Teaching Secondary Mathematics: perspectives on practice (pp. 121–139). https://doi.org/10.4324/9780203165874.

Jumaera, S., Temitope, O., & Rukondo, N. (2024). Optimizing Student Activities and Learning Outcomes through Problem Solving Models in Stoichiometry Material. 1(2), 39–44. https://doi.org/10.37251/jocli.v1i2.1147.

Junius, P. (2008). A case example of insect gymnastics: how is non-Euclidean geometry learned? International Journal of Mathematical Education in Science and Technology, 39(8), 987-1002.

Khadavi, M. R., & Maulana, S. A. (2020). Hubungan Sarana Prasaran Dan Motivasi Terhadap Pencapaian Prestasi Olahraga Siswa Sma Kota Pangkalpinang. Jurnal Muara Olahraga, 2(2), 228–238. https://doi.org/10.52060/jmo.v2i2.312

Kovalenko, O., & Hontarenko, I. (2023). Modern Electronic Educational Technologies in the Process of E-Learning. Educational Challenges, 28(1), 98–112. https://doi.org/https://doi.org/10.34142/2709-7986.2023.28.1.08.

Kurudirek, A. (2023). Illuminating high school students with some interesting non-Euclidean geometries in the plane. International Journal of Statistics and Applied Mathematics, 8(4), 63-68. https://doi.org/10.22271/maths.2023.v8.i4a.1100.

Lagones, J., & Alonso Ishihara, M. (2024). Special noncognitive skill activities (Tokkatsu) during the COVID-19 pandemic in Latin America: the Osoji Japan project as SDG-4 in Peru. Cogent Education, 11(1), 1–21. https://doi.org/10.1080/2331186X.2024.2398858.

Lanawaang, J. J., & Mesra, R. (2024). The Role of Social Media-Based Project-Based Learning Model in Increasing Community Participation in the Process of Forming Environmental Regulations. 5(6), 222–229. https://doi.org/10.37251/ijoer.v5i6.1160.

Lauc, T., Jagodić, G. K., & Bistrović, J. (2020). Effects of multimedia instructional message on motivation and academic performance of elementary school students in Croatia. International Journal of Instruction. https://doi.org/10.29333/iji.2020.13431a.

Liu, Z., Zhang, N., Peng, X., Liu, S., Yang, Z., Peng, J., ... & Chen, J. (2022). Exploring the relationship between social interaction, cognitive processing and learning achievements in a MOOC discussion forum. Journal of Educational Computing Research, 60(1), 132-169. https://doi.org/10.1177/07356331211027300.

Lo, C. K., & Hew, K. F. (2020). A comparison of flipped learning with gamification, traditional learning, and online independent study: the effects on students' mathematics achievement and cognitive engagement. Interactive Learning Environments, 28(4), 464-481. https://doi.org/10.1080/10494820.2018.1541910.

Lomos, C., Seineke, U., Kesting, F., & Luyten, J. W. (2023). The design of incentive systems in digital game-based learning: How primary school children interact with It. Education Sciences, 13(7), 668. https://doi.org/10.3390/educsci13070668.

Madhankumar, S., Prawin Sankar, T. A., Shibin Thomas, S., Brundhavani, G., Paul, P., & Girishkumar, V. (2021). Development of Mobile Application Incorporating Augmented Reality for E-education and Training Systems. IOP Conference Series: Materials Science and Engineering, 1059(1). https://doi.org/10.1088/1757-899X/1059/1/012016.

Mafarja, N., Zulnaidi, H., & Mohd. Fadzil, H. (2022). Using Reciprocal Teaching Strategy to Improve

Physics Students' Critical Thinking Ability. Eurasia Journal of Mathematics, Science and Technology Education, 18(1), 1–14. https://doi.org/10.29333/EJMSTE/11506.
Martin, F., Sun, T., Turk, M., & Ritzhaupt, A. D. (2021). A meta-analysis on the effects of synchronous online learning on cognitive and affective educational outcomes. International Review of Research in Open and Distributed Learning, 22(3), 205-242. https://doi.org/10.19173/irrodl.v22i3.5263.
Matias, C. E. (2020). Do you see the words coming out of that text?: Seeing whiteness in digital text. International Journal of Multicultural Education, 22(2), 14–29. https://doi.org/10.18251/ijme.v22i2.2411.
Mazmurrini, E., Aji, M. P. P., & Suhartono. (2023). The Effect of Using Animation Video on Students' Writing Skills. Eltt, 9(2), 104–110. https://doi.org/10.37251/jolle.v1i2.1063.
Mokoginta, A., & Mokwena, T. (2024). Strategy for Developing the Potential of Sanrobengi Island Tourism Object , Takalar Regency. 1(2). https://doi.org/10.37251/jthpe.v1i2.1184.
Moreno-Guerrero, A. J., Aznar-Díaz, I., Cáceres-Reche, P., & Alonso-García, S. (2020). E-learning in the teaching of mathematics: An educational experience in adult high school. Mathematics, 8(5), 840. https://doi.org/10.3390/math8050840.
Mulligan, J. (2015). Looking within and beyond the geometry curriculum: connecting spatial reasoning to mathematics learning. ZDM Mathematics Education, 47, 511-517. https://doi.org/10.1007/s11858-015-0696-1.
Mulqueeny, K., Kostyuk, V., Baker, R. S., & Ocumpaugh, J. (2015). Incorporating effective e-learning principles to improve student engagement in middle-school mathematics. International Journal of STEM Education, 2, 1-14. https://doi.org/10.1186/s40594-015-0028-6.
Nazarovich, S. T., & Kurudirek, A. (2024). An overview of existing problems in teaching the science "fundamentals of geometry". International Journal of Social Sciences & Educational Studies, 11(2), 91-106. https://doi.org/10.23918/ijsses.v11i2p91.
Noviyanti, M. E. R. Y., & Suryadi, D. I. D. I. (2019). Basic mathematics knowledge of early childhood teachers. Journal of Engineering Science and Technology, 1, 19-27.
Noviyanti, M., & Suryadi, D. (2019, February). Conceptualizing mathematical knowledge for teaching of Indonesian teacher in teaching number sense to early childhood. In Journal of Physics: Conference Series (Vol. 1157, No. 3, p. 032121). IOP Publishing.
O'Rourke, B., Oortwijn, W., & Schuller, T. (2020). The new definition of health technology assessment: A milestone in international collaboration. International Journal of Technology Assessment in Health Care, 36(3), 187–190. https://doi.org/10.1017/S0266462320000215.

Ocktaviani, N., Chisebe, S., & Al-ghzewat, M. (2024). Harmony of Progress : Investigating Discipline , Motivation , and Learning Achievement in Citizen Education. 5(2). https://doi.org/10.37251/jske.v5i2.873.

Oktasari, Y. (2024). Use of Hand Outs in the form of Illustrated Stories to Increase Student Learning Motivation. 1(1), 26–31. https://doi.org/10.37251/jocli.v1i1.1035.

Osman, S. Z. M. (2022). Combining synchronous and asynchronous learning: student satisfaction with online learning using learning management systems. Journal of Education and E-learning Research, 9(3), 147-154. https://doi.org/10.20448/jeelr.v9i3.4103.

Pajk, T., Van Isacker, K., Aberšek, B., & Flogie, A. (2021). Stem education in eco-farming supported by ict and mobile applications. Journal of Baltic Science Education, 20(2), 277–288. https://doi.org/10.33225/jbse/21.20.277.

Rabaza, M & Hamilton, J. (2022). The effect of the euclidean geometry short learning programme using the worked-out examples teaching approach on mathematics teachers' performance. Johme *Journal of Holistic Mathematics Education*, *6*(1), 1-16. https://doi.org/10.19166/johme.v6i1.5264.

Rachmanto, T. B., & Akande , I. O. (2024). Utilization of Information Technology in Increasing the Effectiveness of Citizenship Learning. *Journal of Educational Technology and Learning Creativity*, *2*(2), 217-222. https://doi.org/10.37251/jetlc.v2i2.1140.

Rivera, D. A., Frenay, M., & Swaen, V. (2024). The learning design of MOOC discussion forums: An analysis of forum instructions and their role in supporting the social construction of knowledge. *Technology, Knowledge and Learning*, *29*(2), 585-615. https://doi.org/10.1007/s10758-023-09670-w.

Rosanti, S., Zulkarnain, I., & Rangkuti, L. A. (2022). the Effect of Scramble Method on Students' Achievement in Learning Vocabulary. EXCELLENCE: Journal of English and English Education, 2(1), 10–15. https://doi.org/10.47662/ejeee.v2i1.332.

Ruiz, N., & Gallagher, M. (2025). Rural education imaginaries in digital education policy: an analysis of CONPES 3988 in Colombia. International Journal of Educational Development, 113(February 2024), 103222. https://doi.org/10.1016/j.ijedudev.2025.103222.

Saputra, E., & Fahrizal, E. (2019). The Development of Mathematics Teaching Materials through Geogebra Software to Improve Learning Independence. Malikussaleh Journal of Mathematics Learning (MJML), 2(2), 39–44. https://doi.org/10.29103/mjml.v2i2.1860.

Sari, R., Omeiza, I. I., & Mwakifuna, M. A. (2023). The Influence of Number Dice Games in Improving Early Childhood Mathematical Logic in Early Childhood Education. Interval: Indonesian Journal of Mathematical Education, 1(2), 61–66. https://doi.org/10.37251/ijome.v1i2.776.

Setiawan, A., Nugroho, W., & Widyaningtyas, D. (2023). Development of interactive powerpoint learning media based on information and communication technologies to improve student learning outcomes. Jurnal Pendidikan Dasar Nusantara, 9(1), 75–86. https://doi.org/10.29407/jpdn.v9i1.20056.

Shaame, A. A., Anatory, J. R., Osaki, K. M., & Mrutu, S. I. (2020). Exploring a learning management system as a way to improve students' understanding of geometry in secondary schools. Africa Education Review, 17(4), 17-40. https://doi.org/10.1080/18146627.2020.1868070.

Shida, N., Sharifah, S., Hanifah, H., Norulhuda, N., & Halim, A. (2019). The influence of E-learning towards metacognitive enhancement in mathematical problem solving. International Journal of Emerging Technologies in Learning, 14(20), 165-173. https://doi.org/10.3991/ijet.v14i20.11466.

Siddique, Z. F., Nahar, L., & Mahmood, F. (2025). Autoethnographic Projection of Climate Change Education through Project-Based Learning : Perspectives from Early Career Scholars. 6(1), 38–46. https://doi.org/10.37251/isej.v6i1.1170.

Sudirman, Kusumah, Y. S., & Martadiputra, B. A. P. (2022). Investigating the Potential of Integrating Augmented Reality into the 6E Instructional 3D Geometry Model in Fostering Students' 3D Geometric Thinking Processes. International Journal of Interactive Mobile Technologies (iJIM), 16(06), pp. 61–80. https://doi.org/10.3991/ijim.v16i06.27819.

Sudirman, S., Rodríguez-Nieto, C. A., Dhlamini, Z. B., Chauhan, A. S., Baltaeva, U., Abubakar, A., ... & Andriani, M. (2023). Ways of thinking 3D geometry: exploratory case study in junior high school students. Polyhedron International Journal in Mathematics Education, 1(1), 15-34.

https://doi.org/10.59965/pijme.v1i1.5.

Sudirman, S., Yaniawati, P., Mellawaty, M & Indrawan, R. (2021). Augmented reality application: What are the constraints and perceptions of the students during the covid 19 pendemic's 3D geometry learning process? In Journal of Physics: Conference Series (Vol. 1783, No. 1, p. 012007). IOP Publishing.

Sukestiyarno, Y., Nugroho, K., Sugiman, S., & Waluya, B. (2023). Learning trajectory of non-euclidean geometry through ethnomathematics learning approaches to improve spatial ability. Eurasia Journal of Mathematics Science and Technology Education, 19(6), em2285. https://doi.org/10.29333/ejmste/13269.

Sukri, S., Palinggi, Y., & Petrus Taliabo, L. (2024). The Influence of Health Education on the Level of Knowledge about Hypertension. Jurnal Promotif Reventif, 7(1), 52–57. https://doi.org/10.37251/jhiee.v1i2.1204.

Sun, J. C. Y., Martinez, B., & Seli, H. (2018). Just-in-time or plenty-of-time teaching? Different electronic feedback mechanisms in the online learning environment. Educational Technology Research and Development, 66(2), 283-306. https://doi.org/10.1007/s11423-017-9554-2.

Suroso, S., Solaiman, S. M., & Fanani, A. (2024). The Relationship between the Effectiveness of School Principal Leadership in Improving the Quality of Education. Jurnal Pendidikan Agama Islam Indonesia (JPAII), 5(2), 43–49. https://doi.org/10.37251/jpaii.v5i2.885.

Tachie, S. (2020). The challenges of south african teachers in teaching euclidean geometry. International Journal of Learning Teaching and Educational Research, 19(8), 297-312. https://doi.org/10.26803/ijlter.19.8.16.

Tulabut, R. J., Guzman, R. V., Abaring, P. R. M., Armada, A. P., Ilustre, A. H., & Torda, M. J. T. (2018). Common errors in prepositions committed by grade 9 students: Implications for teachings. TESOL International Journal, 13(3), 113–123.

Tursynkulova, E., Madiyarov, N., Sultanbek, T., & Duysebayeva, P. (2023). The effect of problem-based learning on cognitive skills in solving geometric construction problems: a case study in Kazakhstan. Frontiers in Education, 8, 1284305. https://doi.org/10.3389/feduc.2023.1284305.

Ugwuanyi, C. S. (2022). Developing Sound Knowledge of Basic Science Concepts in Children Using Flipped Classroom: A case of simple repeated measures. Education and Information Technologies, 27(5), 6353–6365. https://doi.org/10.1007/s10639-021-10850-3.

Worachak, S. P., Damnoen, P. S., & Hong, D. A. C. (2023). Analysis of Critical Thinking Skills in Problem-Based Learning and Inquiry Learning Models. EduFisika: Jurnal Pendidikan Fisika, 8(3), 282–293. https://doi.org/10.59052/edufisika.v8i3.29442.

Xie, K., & Ke, F. (2011). The role of students' motivation in peer-moderated asynchronous online discussions. British Journal of Educational Technology, 42(6), 916-930. https://doi.org/10.1111/j.1467-8535.2010.01140.x.

Yang, H., & Romero-Hall, E. (2024). A pilot study exploring interaction and student satisfaction in asynchronous courses in higher education. TechTrends, 68(4), 761-770. https://doi.org/10.1007/s11528-024-00986-7.

Yaniawati, P., Sudirman, S., Mellawaty, M., Indrawan, R., & Mubarika, M. P. (2023). The potential of mobile augmented reality as a didactic and pedagogical source in learning geometry 3D. Journal of Technology and Science Education, 13(1), 4-22. https://doi.org/10.3926/jotse.1661.

Yumiati, Y., Putra, H. D., & Haji, S. (2025). Blended online learning: Students' perception and its effect on learning outcomes abstract algebra. Infinity Journal, 14(1), 65-84. https://doi.org/10.22460/infinity.v14i1.p65-84.

Zawacki-Richter, O., Marín, V. I., Bond, M., & Gouverneur, F. (2019). Systematic review of research on artificial intelligence applications in higher education. International Journal of Educational Technology in Higher Education, 16(1), 1-27. https://doi.org/10.1186/s41239-019-0171-0.

Zulkhi, M. D. (2024). Increasing Students ' Learning Activity Through a Differentized Learning Approach Using the Project Based Learnig Model in Primary Schools. 5(2), 96–107. https://doi.org/10.37251/jber.v5i2.1001.