

Analisis Tipe Kesalahan Siswa dalam Menyelesaikan Soal Matematika Berdasarkan Taksonomi Anderson

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Abstrak

Penelitian ini menganalisis kesalahan siswa dalam menjawab soal dengan level C1-C6 berdasarkan taksonomi Anderson pada pokok bahasan sistem persamaan linear tiga variabel. Penelitian dilakukan dengan pendekatan kualitatif deskriptif dimana data dikumpulkan melalui tes dan wawancara dengan mengambil 30 siswa kelas X di SMAK Santo Yoseph Denpasar. Dalam penelitian ini, siswa diberikan 6 soal matematika, yang kemudian dianalisis kesalahannya dan dilanjutkan dengan mewawancarai siswa untuk menemukan penyebab kesalahannya. Hasil persentase kesalahan siswa berdasarkan tipe kesalahan adalah jawaban kurang lengkap 46,8%, kesalahan teknik 25,5%, kesalahan dalam memahami materi sebelumnya 16,6%, salah menginterpretasikan bahasa 5,5%, data yang disalahgunakan 4,5%, dan solusi yang tidak terverifikasi 1,1%. Dari hasil wawancara ditemukan penyebab siswa melakukan kesalahan yaitu kekurangan waktu, rendahnya motivasi untuk berusaha menyelesaikan soal, tidak melakukan pemeriksaan kembali, ceroboh, kesalahan asumsi yang terjadi akibat cara berpikir siswa yang spontan tanpa melihat kebermaknaan masalah, siswa tidak memahami konsep yang terkait dengan prosedur dalam menyelesaikan masalah, dan kurang perhatian terhadap hasil jawaban yang dibuat. Hasil menunjukkan adanya kesalahan pada tiap level, utamanya pada level yang lebih tinggi yang berdampak pada perlunya memperbanyak latihan soal pada level C4, C5, dan C6 agar pemahaman siswa lebih baik lagi serta meningkatkan motivasi untuk menyelesaikan permasalahan yang diberikan.

Kata kunci: analisis kesalahan, taksonomi Anderson, tipe kesalahan

The Analysis of the Types of Student Error in Solving Mathematical Problems based on the Anderson's Taxonomy

Abstract

This study analyzed student errors in answering questions with levels C1-C6 based on Anderson's taxonomy on the topic of a three-variable system of linear equations. The research was conducted with a descriptive qualitative approach in which the data were collected through test and interviews by taking 30 tenth grade students at Santo Joseph Senior High School Denpasar as the subjects. Students were given 6 math problems, which were then analyzed for errors and followed by interviewing students to find the cause of their errors. The results of the percentage of student errors based on the type of error were incomplete answers at 46.8%, technical errors at 25.5%, errors in understanding the previous material at 16.6%, language misinterpretation at 5.5%, data misuse at 4.5%, and unverified solutions at 1.1%. From the results of the interviews, it was found that the causes of the errors were poor time management, low motivation to try to solve problems, not understanding the problem, not doing re-examination, being careless, errors in assuming that occurred due to students' spontaneous thinking without examining the meaning of the problem, students did not understand the concept that was related to procedures in solving problems, and less attention to the results of the answers made. The results indicated that there were errors at each level, especially at higher levels which have an impact on the need to increase practice questions at levels C4, C5, and C6 so that the students' understanding is better and the motivation to solve the problems given is increased.

Keywords: Anderson's taxonomy; error analysis; type of error

INTRODUCTION

Mathematics is a basic science that can develop human thinking (Rusyani et al., 2022). According to Umay (2003), learning mathematics is vital for human life as it develops the skills in reasoning, interpreting, decision or conclusion making, and problem solving. The essence of learning mathematics is a mental activity to understand the meaning, symbols and relationships which are then applied to real situations (Hamzah & Masri, 2009). Given the importance of mathematics, students should be able to master mathematics well, which can be done through mathematics questions drill that practice students' cognitive ability (Wahyuni & Rahman, 2017).

Aryani et al., (2020) stated that the questions contained in the Final Examination or daily assessments should apply the six levels of Anderson's taxonomy so that they can measure students' cognitive achievement from low level thinking to the highest level of thinking. The cognitive domain in Anderson's Taxonomy consists of two dimensions, which are the cognitive process dimension and the knowledge dimension (Krathwohl, 2002; Wilson, 2016). The knowledge dimension consists of factual, conceptual, procedural, and metacognitive aspects, while the cognitive process dimension consists of C1 (remember), C2 (understand), C3 (apply), C4 (analyze), C5 (evaluate), and C6 (create) (Stanley & Moore, 2013).

Despite the importance of learning mathematics, the reality shows that mathematics is still a scourge for students since it is considered to be a difficult subject (Li & Schoenfeld, 2019; Siregar, 2017). Learning difficulties faced in learning mathematics will result in students being disinterested and bored in learning mathematics (Yeh et al., 2019). Some students tend to work on math problems by memorizing the procedures or steps without understanding the mathematical concepts used, which causes errors in solving math problems.

Errors can be defined as the deviation of the right things that are systematic, consistent, or incidental (Priyani & Ekawati, 2018; Pomalato et al., 2020; Rushton, 2018). In mathematics, errors are the incongruity between the results of calculation with the actual answers. Errors can occur in the process of solving problems, either in the initial, middle, or final steps. Errors in mathematics are often caused by the use of wrong formula which causes the final answer to be wrong as well as the use of procedures that are not fully understood by students (Yang et al., 2011). Movshovitz-Hadar et al., (1987) and Schnepper and McCoy (2013) conducted research in the range of years that were quite distant, which showed that the errors made by students in mathematics are mostly related to algebraic and numerical calculations, such as $7 \times 8 = 54$, $(a - 4 \cdot b - 4 \text{ not } (a - 4) \cdot (b - 4))$, and $\frac{71^\circ}{2} = 35^\circ 5$. The results also indicated that at the completion of story problems, many students are unable to translate the questions into mathematical form or create mathematical models.

The results of interviews with teachers at Santo Yoseph Senior High School Denpasar indicated that students frequently made errors in answering story questions and making mathematical models. The example of the error made by the students was the error in answering math problems that are related to system of inequality. For instance, the students made error in answering math problem (1) $3x-2y+4z=18$, (2) $5x+4y-2z=20$, (3) $2x+y+3z=15$. The results also indicated that there were many students who submitted blank paper during daily assessments. Based on the interviews, it is necessary to analyze the causes of errors made by students. Classroom analysis is generally carried out through examining students' work (Santoso et al., 2017). Then, the teacher can provide corrections or improvements aimed at students' error patterns (Riccomini, 2005). This will have an impact on students' understanding of mathematics on an ongoing basis. Not only beneficial for students, error analysis in mathematics learning will be useful also for teachers since they can find out to what extent students understand the material described (Larrain & Kaiser, 2019). In line with this, Legutko (2008) stated that teachers should analyze the overall mistakes of students, try to understand these mistakes, explain what errors occurred, and find the causes of these errors.

Several research results indicated that there are still many mistakes made by students in solving math problems. Alhassora et al., (2017) found students' errors in answering story questions related to the coordinate system material, which indicates failure in C3. The errors addressed by Alhassora et al., (2017) include (1) failure to understand the problems given, (2) inability to formulate and interpret the right formula, (3) miscalculations, and (4) incongruent conclusions. Mu'minah (2018) also found errors made by students in the C3 level, which include misunderstanding and transformation errors. Purnama (2017) examined students' mathematics C5 level which resulted an average score of 73.4, which was below the minimum completeness criteria.

Considering the importance of learning mathematics for students' thinking ability and the need of classroom analysis that can examine students' work and provide corrections or improvements, further analysis on the errors made by students in solving math problems is needed. Further, taking into account the statement by Legutko (2008) that teachers should analyze the overall mistakes of students, try to understand these mistakes, explain what errors occurred, and find the causes of these errors, this analysis is urgently needed. In addition, there has been no research that has conducted an error analysis at the level of C4 to C6 questions with the material of a three-variable system of linear equations.

Therefore, this study arose to provide a more in-depth analysis on the types of errors made by students in answering mathematics questions based on Anderson's taxonomic cognitive domain. The factors that cause these errors were also analyzed so that later the teacher can minimize the errors made by students. With regards to the results of the initial interviews with the teachers at Santo Yoseph Senior High School Denpasar, the school was selected to be the setting of the study. It is considered appropriate considering the results of the interviews that showed students' frequent errors in answering story questions and making mathematical models. This study is needed so that students will be able to correct their mistakes through reflection and teachers can provide the needed guidance to help the students.

METHOD

The method used in this research is descriptive qualitative (Sugiyono, 2015). Students' errors in solving math problems with Anderson's Taxonomy cognitive domain were analyzed through written test and interviews. The research subjects were 30 students of the X grade at Santo Yoseph Senior High School Denpasar. Students were given 6 essay questions. Then, the results of their work were analyzed and grouped with regards to the type of error suggested by Schnepfer and McCoy (2013) and Movshovitz-Hadar et al., (1987). Then, the subjects were selected for the interview. The selected subjects were students who had good communication skills so that the expression of thinking processes when solving problems could be conveyed properly. Subject selection was done by adjusting the objectives of the researcher so that the subjects could provide the needed data for the study.

The instrument used in this study were an essay test and interview guide. The essay test consisted of 6 questions and it was used to show how students solved the math problems and put their thoughts in answering the problems given. This test could be used to determine the type of students' errors. Based on the students' answers, the students who answered incorrectly would be grouped according to the type of error, which then led to an interview on the selected subject that was conducted based on the interview guide. The interview was conducted as a form of methodological triangulation to ensure the reliability of the data. As suggested by Heale and Forbes (2013), methodological triangulation is a triangulation that includes two or more sets of data collection in the same research design, which in this case were test and interview.

The type of student error was adapted according to Schnepfer and McCoy (2013) and Movshovitz-Hadar et al., (1987) which consisted of (1) incomplete answers, (2) data misuse, (3) technical errors, (4) errors in understanding the previous material, (5) language misinterpretation, (6) theorems and definitions misuse, and (7) unverified solutions.

Errors in the forms of incomplete answers refer to questions that are answered but do not have all the solutions and do not provide the conclusions required by the problems. The indicators of data misuse include (a) using data from unstated information, (b) ignoring some data needed to find a solution and adding irrelevant data, (c) stating explicitly (e.g. “must be proven”, “found”, “calculated”) as a requirement when not needed in problem solving, (d) specifying certain inconsistent information (e.g. using the height of a triangle to solve problems about the median), (e) using terms that do not match the information provided, and (f) using the numeric value of one variable for another variable.

Technical errors include calculation errors, errors in extracting data from tables, errors in manipulating basic algebraic symbols, and other errors in writing algorithms. Then, as the name suggests, errors in understanding previous material occur due to not understanding the concept of the previous material.

Language misinterpretation refers to mathematical errors that are related to the translation of mathematical facts described into other possible languages. The indicators of this error are (a) translating natural language into mathematical terms or equations that are not appropriate from the explanations given, (b) designing a mathematical concept with symbols of other mathematical concepts, and (c) misinterpreting the graph into mathematical terms or vice versa.

Theorems and definitions misuse refers to errors in the use of principles, rules, theorems, and definitions, such as (a) using theorem not under appropriate conditions, (b) applying the distributive property to nonconforming operations, and (c) citing inaccurate definitions, theorems, and formulas.

Finally, errors related to unverified solutions is indicated by the final result that is not a solution to the stated problem even though the steps conducted in answering the questions are correct. This is due to the fact that students do not re-examine or verify the solutions that have been made.

Data analysis was performed by using the Miles and Huberman model, which include data reduction, data display, and conclusion drawing or verification. The data from the test and interview firstly went through data reduction in which the information were processed and reduced in order to have the needed, important data related to the students’ errors in answering math problems. Then, the data were classified or categorized in the data display procedure, which were then drawn to a conclusion in the conclusion drawing stage.

RESULTS AND DISCUSSION

Based on the results of the essay test, several types of errors were found at each level of the questions. The research subjects interviewed were S1, S2, S3, S4, S5 and S6. Each of them had errors at the C1 to C6 level that were based on the type of errors according to Schnepfer and McCoy (2013) and Movshovitz-Hadar et al., (1987). Subjects for the interview were taken by noting that the students chosen made the most types of errors, with unique errors, or errors that occurred repeatedly. The results for the percentage of error types at each level of the questions can be seen on Table 1.

Table 1 Frequency Distribution of Data Analysis Regarding the Student Error Types based on Movshovitz-Hadar and Schnepfer

Error Types according to Movshovitz-Hadar and Schnepfer	Percentage of Error Types at Each Level of Mathematical Problems						Total
	C1	C2	C3	C4	C5	C6	
Incomplete answers	18.9%	10%	4.5%	2.2%	4.5%	6.7%	46.8%
Data misuse	-	4.5%	-	-	-	-	4.5%
Technical error	-	11.1%	11.1%	2.2%	-	1.1%	25.5%
Error in understanding the previous material	10%	2.2%	-	-	3.3%	1.1%	16.6%
Language misinterpretation	2.2%	-	-	2.2%	1.1 %	-	5.5%

Error Types according to Movshovitz-Hadar and Schnepfer	Percentage of Error Types at Each Level of Mathematical Problems						Total
	C1	C2	C3	C4	C5	C6	
Theorems and definitions misuse	-	-	-	-	-	-	-
Unverified solutions	-	-	1.1%	-	-	-	1.1%
Total	31.1%	27.8%	16.7%	6.6%	8.9%	8.9%	100%

Based on Table 1, it can be seen that errors in the form of incomplete answers appeared most frequently with the percentage of 46.8%. It is then followed by technical error with 25.5%, error in understanding the previous material with 16.6%, language misinterpretation with 5.5%, data misuse with 4.5%, and unverified solutions with 1.1%. The results also indicate that there was no error related to theorems and definitions misuse. For more detailed information, Figure 1 shows S1's answers in answering the question for C1 level.

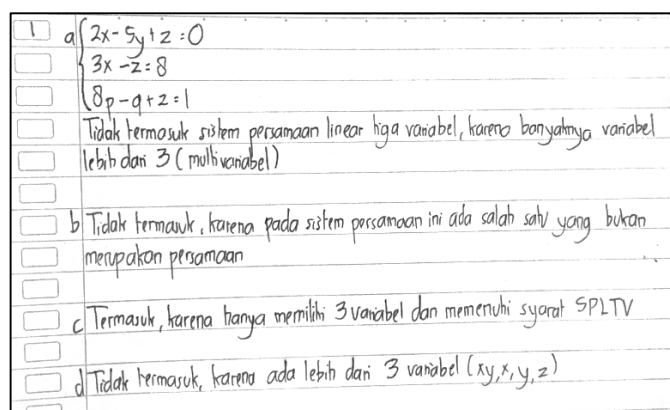


Figure 1. The Answer of S1 for C1 question

Based on the results shown on Figure 1 along with the results of the interview with S1, it was found that the types of errors made were (i) incomplete answers and (ii) errors in understanding the previous material. Incomplete answers could be observed from S1's answer of point c). The answer was incomplete because S1 did not write down the Linear Equation in Three Variables requirements. Even though these requirements were very important to be written down, S1 ignored them. Then, S1 made a mistake in understanding the previous material. Previous material that was not understood was the definition of the equation which caused S1 to experience an error in answering question b). On the interview, S1 stated that he forgot that an equation includes an "=" sign. But after that, S1 was even able to add the existing sign to the inequality. This means that a spontaneous thought process occurred, which led to the error.

The results of the interview also indicated that S1 understood the beginning of the problem and understood what must be answered. Furthermore, S1 could explain the methods that should be used to answer question C1. However, in the process of answering, S1 seemed to have lack of control over what the subject had in mind, therefore an error occurred. This can also be because S1 had not used the thought process optimally because the concepts stored in S1 memory were not well connected. This statement is also in accordance with Wibawa (2014) who stated that errors can occur because students' thinking processes are not optimal and the concepts they already have are not well connected. Similarly, Pomalato et al. (2020) also suggest that this error can happen due to the students' error processing skill that makes them use rules correctly but make mistakes in the computation. In addition, Santoso et al. (2017) state that it is a common error that appears in mathematics.

Then, the cause of errors in understanding the previous material was that students could not construct new knowledge with the knowledge they have. This error involves

misunderstanding and implementing inappropriate resolution procedures. According to Elbrink (2008) this type of error indicates that students do not understand the concepts associated with procedures. As a result, students do not have an understanding of why or how a procedure works, therefore students do not recognize the importance of implementing and calculating procedures correctly. Bethany (2016) revealed that there are several ways that can be done to prevent these mistakes, namely giving students plenty of time to explore and discover new mathematical concepts in a way that helps them see and understand the problem. The reasons behind is that a mathematical formula will help students understand, form connections, and maintain concepts. Then, there is always more than one way to solve math problems. By teaching or exploring concepts in multiple ways and from multiple angles, teachers provide students with a richer mathematical environment, and this would enable deeper understanding. Some students may find one method easier, while other students prefer a different method.

More errors were found on S2's answer on answering C2 level questions. The answer of S2 is shown on Figure 2.

2) Dik: $3x - 2y + 4z = 18$ (i)
 $5x + 4y - 2z = 20$ (ii)
 $2x + y + 3z = 15$ (iii)

eliminasi:
 $3x - 2y + 4z = 18$ | $\times 2$ | $6x - 4y + 8z = 36$
 $5x + 4y - 2z = 20$ | $\times 1$ | $5x + 4y - 2z = 20$
 \hline
 $11x + 6z = 56$ (iv)

eliminasi:
 $5x + 4y - 2z = 20$ | $\times 1$ | $5x + 4y - 2z = 20$
 $2x + y + 3z = 15$ | $\times 4$ | $8x + 4y + 12z = 60$ -
 \hline
 $-3x \quad -14z - 40$ (v)

$3x + 10z = 46$ | $\times 3$ | $21x + 30z = 144$
 $-3x - 14z = -40$ | $\times 7$ | $-21x - 98z = -280$ +
 \hline
 $-68z = 124$
 $z =$

Figure 2. The Answer of S2 for C2 question

From the analysis of the answers and interviews with S2, the type of mistakes made were (i) incomplete answer, because S2 could not solve the problem in C2 to reach the result of $a^2 + b^2 + c^2$, (ii) technical error, S2 made a technical error because in the process of elimination a '+' sign should have been written but S2 wrote a '-' sign, but the result was an addition, (iii) data misuse, in this type of error S2 added foreign data to the solution whereas the equation used did not exist in the problem and did not exist from the previous elimination process.

According to the results of the interview, S2 seemed less focused because the subject was in a hurry to solve the other problems. Thus, the factor that cause this error is poor time management. This can be prevented by giving students various kinds of practice questions in the form of stories or exact and give a time limit for each question. Thus, students will be trained to solve questions on time. Schnepfer and McCoy (2013) also emphasized in his research that students who do not complete their answers must take part in re-learning by re-emphasizing concepts related to questions. The emphasis that must be considered in learning is the missing answer to each question. The results also indicated that S2 also did not re-check the answer so that an error occurred. In general, the concept of Linear Equation in Three Variables completion has been understood by S2, but it needs to be retrained regarding skills in choosing equations that are easier to eliminate.

Errors that occurred on C3 level are reflected on the answers of S3, which can be seen on Figure 3.

Based on the analysis of the answers and interviews with S3, the types of errors found in the answer for C3 was (i) incomplete answer because the answer was not accompanied with information about the analogy of variables in the mathematical model that were made. The

incomplete answers can also be seen because S3 was not able to complete the answer until a result was obtained. Another error that occurred was (ii) technical errors since there were calculation errors during the elimination process.

Figure 3. The Answer of S3 for C3 question

The incomplete answer occurred because S3 has experienced a calculation error at the beginning of solving the problem which resulted in an error in the next process. Lack of arithmetic operation skills was also the cause of errors and students' lack of motivation to keep trying to find answers. In addition, S3 also did not re-check the answer. This results indicated congruity with the previous studies conducted by other researchers. It is in line with the study by Alhassora et al., (2017) that found students' errors in answering story questions related to the coordinate system material, which indicates failure in C3. It is also in line with the results of study by Mu'minah (2018) who also found errors made by students in the C3 level, which include misunderstanding and transformation errors. Moreover, the errors addressed in S3 that are related to C3 level is also in line with the one found by Alhassora et al., (2017) as it includes miscalculations.

Similar to S2, S3 also did not recheck his answers before submitting the paper. Therefore, the cause of this error was carelessness, lack of attention to the results of the answers made, and lack of practice questions related to algebraic operations or calculation operations. Elbrink (2008) states that the solution that can be done to reduce calculation errors is to include a list of errors into the routines and procedures in each hour of mathematics lessons. This allows students to self-assess and identify repeated mistakes and errors in their work.

The next errors are related to C4 level, which is reflected by the answers calculated by S4. S4's answer of the C4 level question can be seen on Figure 4.

Figure 4. The Answer of S4 for C4 question

The results of the answers and interviews showed that S4 has made a mistake in solving C4 level question. Based on the characteristics found, the types of errors that were committed were (i) language misinterpretation and (ii) incomplete answers.

The error in interpreting the language occurred because S4 was unable to translate the story problem into mathematical form correctly. This was also supported by the translation of mathematical terms or equations that did not match the explanation given verbally. S4 stated that he was confused

about the "other number" phrase. It can also be caused by not focusing on the main problem. From the very beginning there were errors that caused errors on an ongoing basis. Thus, S4 could not understand the problem well so that an error occurred in the mathematical model created. The error of language misinterpretation here is in line with the results of the studies by Alhassora et al., (2017) who found failure in understanding the problems given and Mu'minah (2018) who also found errors related to misunderstanding.

Errors in interpreting language were found when students translated the form of the story problem into a mathematical model. Based on the results of interviews and discussions, the cause of this error occurred because students did not understand the problems given and students had difficulty imagining the problems given to real life. This error could also occur due to an incorrect assumption. Errors in assumptions made occurred due to students' spontaneous thinking without examining the meaning of the problem, meaning that students do not control what is being thought and what is being done (Wibawa, 2014). This error occurs when students incorrectly associate math problems that use the same symbols. Students try to make meaning in the patterns of symbols and mathematical signs they see in front of them rather than trying to understand what they are actually doing. The search for patterns in these symbols leads to misinterpretation, which in turn results in errors in mathematical modeling. Misinterpreting language can be minimized by defragmenting students' thought processes (Wibawa, 2014). Furthermore, Pomalato et al. (2020) also support the theory of how this error can happen as they suggest that problem in determining the relationship between the solutions obtained with the questions asked can lead to error. Defragmenting students' thought processes is done by asking questions that can improve students' thinking processes and linking concepts to solve problems correctly.

The incomplete answers also became an error for S4 mistake in translating the story problem into math equations. If the variables x , y , and z were given earlier information, it would help S4's thinking process to create a mathematical model. It seemed clear that the concepts and understandings that are understood were not interconnected, composed, and formed in a unity to solve the given problem. As the most frequent error, S4, just like S2 and S3, also made incomplete answer, which according to Schnepper and McCoy (2013) can be solved by making the students take part in re-learning by re-emphasizing concepts related to questions.

The next errors are related to C5 level, which is reflected by the answers calculated by S5. S5's answer of the C5 level question can be seen on Figure 5.

5 B. Arif = 1 27 + U. Arif
Ibu Arif = (27 + U. Arif) - 2
Bapak Arif + Ibu Arif + Arif = 133
Dit: Arif + Ibu arif
↳ Jawaban yang menurut saya benar adalah kedua jawaban, tetapi jawaban boy lebih pasti dan akurat. Dan jawaban sita didapatkan dengan cara coba, yang jika tidak tepat memilih angka yg dicoba bisa salah. Jawaban boy bisa didapat dengan cara

Figure 5. The Answer of S5 for C5 question

The types of errors obtained in question C5 based on the results of the answer analysis and the interview was (i) incomplete answer, because it did not provide a reason why choosing Boy's answer was more accurate, then S5 also did not write down the answer correction to Sita's answer. Then, (ii) language misinterpretation also occurred because S5 was unable to understand the problems given to Sita and Boy. What is known in the questions was not taken into consideration in choosing the more appropriate answer between Boy and Sita. Then, S5 also did not notice that the total age of Arif, Father, and Mother was 133 years. As a result, the mistake of translating the real problem into real life

resulted in an error in answering question C5. Then the third type of error was (iii) error in understanding the previous material. This was obtained from the interview results because S5 stated that he did not understand the method of substitution and only understood the method of elimination. Therefore, because Boy and Sita's answers have the same final result, S5 concluded that both answers were correct.

The error found on the C5 level is in line with the study by Purnama (2017) who also found students' mathematics errors on C5 level. In this study, the error occurred because the mechanisms that were understood by S5 to construct real problems into a mathematical model were not sufficient. The errors that occurred were also caused by the construction of the problem representation that was not appropriate, giving rise to justification for both answers. The subject also tended to pay more attention to the results rather than to the completion process. Answers that were not being re-examined also triggered errors, as evidenced by S5 not realizing the sentence that "the total age of the three of them (Arif, Father and Mother) is 133 years." The concepts that were previously understood were also not well connected, S5 only focused on one way that was understood without paying attention to other ways that could be used to solve SPLTV. Therefore, taking the theory suggested by Schnepfer and McCoy (2013) and Movshovitz-Hadar et al., (1987) into account, the errors that were made were (i) incomplete answers, (ii) errors in understanding the previous material, and (iii) language misinterpretation.

Errors that occurred on C6 level are reflected on the answers of S6, which can be seen on Figure 6.

6. Suho, Cha Eun Woo, Jihoon, dan Rowoon pergi ke toko alat tulis untuk membeli buku tulis, penggaris, dan correction. Suho membeli 5 buku tulis, 2 penggaris, dan 3 correction. Cha Eun Woo membeli 4 buku tulis, 3 penggaris, dan 1 correction. Jihoon membeli 3 buku tulis, 5 penggaris, dan 2 correction. Rowoon membeli 2 buku tulis, 3 penggaris, dan 4 correction. Total pembelian Suho adalah Rp 43000, total pembelian Cha Eun Woo adalah Rp 35000, total pembelian Jihoon adalah Rp 42000, dan total pembelian Rowoon adalah Rp 43000. Suho membayar dengan total Rp 200.000, berapakah kembalinya? Jawab:

Buku tulis = x
 Penggaris = y
 Correction = z

$$\begin{aligned} 5x + 2y + 3z &= 43000 \text{ (i)} \\ 4x + 3y + z &= 35000 \text{ (ii)} \\ 3x + 5y + 2z &= 42000 \text{ (iii)} \\ 2x + 3y + 4z &= 43000 \text{ (iv)} \end{aligned}$$

Eliminasi (i) dan (ii)

$$\begin{array}{r} 5x + 2y + 3z = 43000 \quad \times 2 \\ 4x + 3y + z = 35000 \quad \times 2 \\ \hline 10x + 4y + 6z = 86000 \\ 8x + 6y + 2z = 70000 \\ \hline 2x - 2y + 4z = 16000 \end{array}$$

Eliminasi (ii) dan (iii)

$$\begin{array}{r} 4x + 3y + z = 35000 \quad \times 1 \\ 3x + 5y + 2z = 42000 \quad \times 2 \\ \hline 4x + 3y + z = 35000 \\ 6x + 10y + 4z = 84000 \\ \hline -2x - 7y - 3z = -49000 \end{array}$$

Eliminasi (iv) dan (v)

$$\begin{array}{r} 2x + 3y + 4z = 43000 \quad \times 1 \\ 7x + 7z = 77000 \quad \times 1 \\ \hline 2x + 3y + 4z = 43000 \\ 7x + 7z = 77000 \\ \hline -5x - 4z = -34000 \end{array}$$

Eliminasi (v) dan (vi)

$$\begin{array}{r} 7x + 7z = 77000 \quad \times 1 \\ 11x - z = 49000 \quad \times 7 \\ \hline 7x + 7z = 77000 \\ 77x - 7z = 343000 \\ \hline -70x = -266000 \end{array}$$

Figure 6. The Answer of S6 for C6 question

In answering the questions with the highest level, correct thinking skills are needed. In addition, the mastery of concepts is also needed by students in order to answer this type of questions. Based on the results of the analysis of answers shown on Figure 6 along with the results of the interviews, the types of errors found were (i) incomplete answers, (ii) technical errors, and (iii) errors due to not understanding the previous material.

Incomplete answers can be observed from creating problems that are not according to the instructions. S6 stated that the problem was made wrong, and it was necessary to add 1 more person to the purchase transaction so that it can be solved by elimination or substitution. Then because the questions were not resolved, it was concluded that the answers given were insufficient and incomplete. Incomplete answers is the type of error that most frequently occurred as S1, S2, S3, S4, and S5 also made the same errors. This happened due to poor time management. This can be prevented by giving students various kinds of practice questions in the form of stories or exact and giving a time limit for each question. Thus, students will be trained to solve questions on time. Then Schnepfer and McCoy (2013) emphasized in his research that students who do not complete their answers must take part in re-learning by re-emphasizing concepts related to questions. The emphasis that must be considered in learning is the missing answer to each question.

A technical error occurred in the process of eliminating equations (iv) and (v), which should have been marked with a "+" sign, but S6 made a "-" sign. Then the error due to not understanding the previous material can be observed from the problems made. S6 could not distinguish what kind of problem could be solved by elimination and substitution methods. S6 only focused on many equations obtained on the problem, but did not understand the problems that were made well. This is very closely related to the concept of the linear equation in three-variable system material, because if one would like to make the right questions, of course the concept of learning materials must be well mastered. This error was also made by other students being interviewed. Technical errors are a type of error that always occurs. This type of error was always found in students who did wrong calculations such as addition, subtraction, multiplication, and division. Based on the results of interviews and answer analysis, the cause of this error was carelessness and lack of attention to the results of the answers made. Elbrink (2008) stated that a solution that can be done to reduce calculation errors is to enter a list of errors into routines and procedures at each math lesson. This allows students to assess themselves and identify mistakes and mistakes that are repeated in their work.

The researchers suspect that the error occurred because the subject did not focus on the instructions given on the C6 level questions. S6 only focused on one point, not as to the whole problem which made the problem was not in accordance with the instructions. Carelessness in calculations also caused technical errors, where S6 felt rushed to complete calculations so the subject was careless in doing calculations. The concept that was mastered was also not well connected. Researchers also suspect that the subject was not used to making questions and was not given high-level question exercises, so that the skills and reasoning were lacking to make appropriate problems. To reduce or prevent this type of error, students must be calm in answering questions. Encourage them to calm down and pay attention to what they are doing. Both numbers and story questions, circling important information will help students know what to do (Bethany, 2016). Circling the key information in each question will help students think of strategies and ensure that students do not forget what to look for or count in the questions, so that misuse of data can be avoided. Then using a piece of graph paper to solve the problem can be a very simple solution for students who are not used to neat writing. Graph paper allows students to write answers neatly, correctly and prevents unnecessary mixing of data.

Based on the results of the analysis and interviews, various types of errors were found in questions C1 to C6. These types of errors include 46.8% incomplete answers, 4.5% data misuse, 25.5% technical errors, 16.6% errors in understanding the previous material, 5.5% language misinterpretation, and 1.1% unverified solutions. The incomplete answers were the errors that had the highest percentage among other types of errors. From the results of interviews and answer analysis, it can be seen that the causes of incomplete student answers were the poor time management, low motivation to try to solve questions, not understanding problems, not doing re-examination, and being careless.

CONCLUSION

The types of errors found in math problems with C1 to C6 levels ranges in percentage. The most frequent one that occurred was incomplete answers with 46.8%. It was followed by technical errors with 25.5%, errors in understanding the previous material with 16.6%, language misinterpretation with 5.5%, data misuse with 4.5%, and unverified solutions with 1.1%. The causes of the types of errors found were poor time management, low motivation to try to solve the problem, not understanding the problem, not doing re-examination, being careless, assuming errors that occurred due to students' spontaneous thinking without examining the meaning of the problem, not understanding the concepts related to procedures in solving problems, and less attention given to the results of the answers made.

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